PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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Monday, 7*th March* 1864.

PROFESSOR KELLAND, Vice-President, in the Chair.

The following Communications were read :—

1. On the Gods of the Rigveda. By John Muir, D.C.L., LL.D.

After some preliminary remarks on the common origin of the Indians, Greeks, and Romans,—on the expectation thereby raised that we should find in the earliest literatures of these nations some remains of the primeval mythology which their ancestors must originally have possessed in common,—on the partial fulfilment of this expectation by an examination of these literatures,—and on the greater light thrown by the Rigveda than by any other monument of ancient poetry on the genesis of mythology,—the author adverts to the various theories of creation which would naturally be formed by simple men in the earlier ages of the world, to the manner in which the various great phenomena of nature would come to be ascribed to different deities, and to the diverse aspects in which the grander objects of creation, such as heaven and earth, were viewed, sometimes as inanimate, sometimes as animated and divine. The chief deities mentioned in the Rigveda are then passed under review, and their most remarkable characteristics are described ;— viz. *Dyaus* and *Prithivî* (Heaven and Earth), *Aditi* and the *Adityas*, *Varuna* and *Mitra*, *Indra*, *Vâyu*, the *Maruts*, *Rudra*, *Vishnu*, *Tvashtri*, *Agni*, the *Asvins*, *Soma*, *Yama*, and the various goddesses, *Ushas, Sarasvatî*, &c. &c. The writer then sums up the instances of well established coincidence between the Indian and the classical deities, which he considers to be few in number; and concludes by noticing how the most prominent gods of the Rigveda lost much of their importance in the later mythology of India, while two of the deities who hold only a subordinate rank in the Veda—viz. *Vishnu* and *Rudra*—were afterwards exalted to the highest position.

2. On the Diffraction Bands produced by Double Striated Surfaces. By Sir David Brewster, K.H., F.R.S.

Having observed a series of serrated fringes in examining the colours produced by the fibres of the crystalline lens of fishes, the author was led to imitate them by the combination of grooves upon glass and steel surfaces, or of grooves taken from these surfaces upon isinglass or gums.

The interference bands thus produced were serrated or rectilineal, sometimes parallel and sometimes at right angles to the direction of the grooves, and varying in their magnitude and character according as they were exhibited on the colourless image, or on the diffracted spectra, or as they were produced at different angles of incidence, or at different distances of the grooved surfaces, or by different numbers of reflexion, or by different numbers and combinations of refracting and reflecting surfaces.

The grooves on glass employed by the author were executed by the late Mr George Dollond, and those on steel, varying from 315 to 10,000 in an inch, by the late Sir John Barton.

3. An Essay on the Theory of Commensurables. By Edward Sang, Esq.

The subject of this essay may be described as an application of the Theory of Number to Geometry, its principal or characteristic problem being to determine under what conditions the sides or surfaces of figures may be represented by integer numbers.

Like all other inquiries into the properties of integers, it is rather speculative than practical, and yet, perhaps on that very account, is more apt to engross the attention of its cultivators than almost any other department of pure mathematics. It seems, indeed, to be of very little moment whether we can demonstrate that the sum of two cubes never can be a cube; and yet one after another of the most eminent mathematicians have tried, and, though foiled, have again and again essayed the proof of Fermat's negation. So it may also appear to be a matter of indifference whether or not we can construct a four-sided figure which may have its four sides, and also its two diagonals, all integer multiples of the linear unit; and yet such inquiries present to the mathematician attractions sufficiently powerful to balance those of more practical investigations.

Nor is the labour bestowed on the cultivation of such subjects altogether or in any degree lost, since the various branches of science are so interwoven, that we cannot improve our acquaintance with one without augmenting our knowledge of those allied to it.

The first part of the paper is occupied with the subject of the orthagonal trigon, and is a collection of previously known propositions, the novelty, if any, being in the arrangement.

The second powers of numbers form the only exception to Fermat's Theorem; the sum of two squares may be a square number; that is to say, the equation

$$a^2+b^2=c^2$$

is possible in integer numbers; or, in other words, the altitude, the base, and the hypotenuse of a right-angled trigon may all be expressed in integers.

Among the remarkable properties of these Pythagorean numbers, as they are often called, are that, when in the lowest terms, one or other of the two sides is divisible by 3; that one or other of the two sides is divisible by 4; and that one of the three is divisible by 5. These three propositions are all exemplified in the well-known solution,

$$3^2+4^2=5^2.$$

It is also a very singular property, that the hypotenusal number can never be a multiple of 7, of 11, of 19, or, in general, of any prime number of the form $4n-1$, unless the other also be so; and thus that no prime number of that form can ever be a divisor of the hypotenuse when the trigon is in its lowest terms.

And as a companion, we have this other property, that every prime number of the form $4n+1$, and every product of such prime factors, may be the hypotenuse of a right-angled trigon.

This part of the subject is completed by a table showing every form of right-angled trigon, having its sides expressed in integer numbers, with the hypotenuse under 1000.

In the second part of the paper, the properties of the angles of such right-angled trigons are investigated: to these angles the name *muarif* is given, and their values are entered opposite each of the trigons in the above-mentioned table. Muarif angles are defined to be those which have their sines and cosines rational; and it is shown that the sines and cosines of the sum, or difference of two muarif angles, are also rational, this property being analogous to the arithmetical proposition, that the product of the sum of one pair of squares by the sum of another pair, is also the sum of two squares.

This property of muarif angles is then applied to the demonstration of various theorems, and to the solution of several problems. In the first place, it is shown that if a trigon be constructed with two of its angles muarif, the three sides, the three altitudes, the radius of the circumscribing circle, and the radii of the four circles of contact, are all commensurable, while the area also is commensurable with their squares.

This proposition is then extended thus: that if at the ends of any line assumed as a base, muarif angles (in any number) be made, the sides of these extended indefinitely intercept segments, which are all commensurable with the base, and include areas which are all commensurable with the square of the base.

And, farther, that the same property is extended to the sides of all muarif angles made at any of the intersections of the above-mentioned lines.

Also, it is shown that if a straight line be drawn to touch a circle, and if at the point of contact any number of muarif angles be made, if the extremities of the chords thus formed be joined, and if tangents be applied at those extremities, all the lines being continued indefinitely, then all the intercepted distances are commensurable with the diameter, and all the areas with the square of the diameter.

It is then shown how to construct a muarif angle which may approximate with any required degree of precision to a given angle, and thence how to find a rational trigon approximating to a given shape.

The solution is also extended to this more general problem: having given any polygon inscribed in a circle, to find another approximating to it, and having its sides commensurable with the diameter, and its area with the square of the diameter.

Afterwards, it is shown how to construct a trigon having its sides and the lines bisecting its angles all rational.

In the third part of the paper the construction of polygons having their sides, and also the ordinates of their corners integer, is discussed.

The only regular polygons which can be used to cover surface are the *trigon*, the *tetragon*, and the *hexagon*. Of these the regular tetragon or square is the one in common use for the measurement of surface; but, viewing the matter abstractly, we may as well measure surface by triangular inches as by square inches. Since the regular hexagon contains exactly six regular trigons, it follows that, as far as the doctrine of commensurables is concerned, there are only two possible systems of surface measurement,—viz., That with the square, and that with the equilateral trigon as the superficial unit.

The fourth section of the paper is occupied in discussing the trigonal system of measurement. Just as the right-angled trigon is the guide to the theory of tetragonal commensurables; the trigon having an angle of 120° is the guide to the theory of trigonal commensurables. The leading property of such a trigon is, that the square of the subtense exceeds the squares of the two containing sides by their rectangle; but this is an enunciation in tetragonal language: stated appropriately it is this, that the equilateral trigon constructed on the subtense, is equivalent to those on the two sides together with the original trigon; for in this system, the surface of an equilateral trigon represent the second power of a number, and that of a trigon of 120° (or 60°) the product of two numbers. The arithmetical representative of such a trigon is

$$a^2 + ab + b^2 = c.$$

When the trigon is in its lowest terms, that is, when a, b and c have no common divisor, c cannot be divisible by 2, by 3, by 5, by 11, or in general by any prime number which is not of the form $6n + 1$, and conversely, it is shown that every prime number of the form $6n+1$, and every product of such primes, may represent the subtense of 120°, the other sides being rational.

It is worthy of remark, as a notable relation between the properties of numbers and those of surface, that 4 and 6 are the only moduli which separate prime numbers into two classes; all primes, with the exception of 2, being of one or other of forms $4n-1$ and $4n+1$; while, excepting 2 and 3, all belong either to the form $6n=1$ or to $6n+1$. Four squares may lie round a point, and the form $4n+1$, includes all the hypotenuses of right-angled trigons; six equilateral trigons lie round a point, and the form $6n+1$ contains all the subtenses of 120° or of 60°.

A list is given of trigons of 120°, in the lowest terms, of which the subtense does not exceed 1000, accompanied by the value of the smaller angle.

This leads to the recognition of *muarif angles* of the trigonal system, possessing properties analogous to those of the common or tetragonal system.

Thus, if at the extremities of any base trigonal muarif angles be made, all the segments into which the sides of these angles cut each other are commensurable with the base, and all the areas with its equilateral trigon.

And similarly, if at the point of contact of a straight line and circle, trigonal muarif angles be made, if the extremities of the chords be joined, and if tangents be applied at those extremities, all the segments so formed are commensurable with the side, and all the areas with the area of the circumscribed regular trigon; and it may be remarked that in the trigonal system, the inscribed trigon is commensurable with the circumscribed, whereas the inscribed square is incommensurable with the circumscribed.

The existence of these two distinct, yet analogous systems of muarif angles, naturally suggests the inquiry, whether there may not be other systems as well.

The fifth section of the paper treats of muarif systems in general; it shows that if a trigon be constructed with sides, proportional to any three integers whatever, its angles belong to a system of muarif angles possessing properties analogous to those of the two preceding systems, the assumed trigon, or any other one of the system, becoming the unit of surface.

It is also shown that no angle of one can be equal to that of any other muarif system, with the exception of 180° which belongs to all systems.

The general character of a muarif angle is, that its cosine is rational; the value of its sine may or may not involve the square root of an unsquare number; if the sine be rational, the angle belongs to the tetragonal system; but if otherwise, the irreducible surd involved in the expression for the sine becomes the *modulus* or *mastar* of the system, and all angles having the same irreducible surd in the values of their sines, the cosines being rational, belong to the same system.

The modulus of the common or tetragonal system is thus $\sqrt{1}$, that of the trigonal system is $\sqrt{3}$; while the modulus of any other system is the irreducible surd in the common expression for the area of any trigon belonging to it.

The sixth and last section of the paper contains a few miscellaneous propositions. The first group of problems are cases of this general one, "To construct a trigon, of which the three sides and the lines dividing one or more of the angles into equal parts may be all commensurable."

When only one angle is proposed to be divided, or when two angles are to be divided, we can assume these as the proper multiples of muarif angles of any system whatever; but when the three angles are to be divided, we find ourselves restricted in the choice of the system.

Thus, if we wish that the lines bisecting each of the angles be rational, we must use the common or the tetragonal system, because the half of 180° belongs to it. While, if we wish that the lines trisecting the three angles be all rational, we must take the trigonal system, because the third part of 180° is among its angles.

And it is remarkable that we cannot construct a trigon of which the sides and the lines dividing its angles into any other number of equal parts than two and three, may be all rational, because no aliquot part of the half revolution except the half and the third can belong to any muarif system.

P.S.—Since the paper was read, a treatise by Professor Gill of New York, on the "Application of the Angular Analysis to the

Solution of Indeterminate Problems," has been shown to me. In this excellent work the properties of the muarif angles of the tetragonal system are given; and as there is no hint concerning the higher systems of these angles, my impression is confirmed that this part of the subject is entirely new.

4. On Superposition, No. II. By Professor Kelland.

5. On Centrobaric Bodies. By Professor W. Thomson.

(*Abstract by Professor Tait.*)

This is an abstract of an investigation which will be published in full in "Thomson and Tait's Natural Philosophy." It contains the application of Green's wonderful results regarding the potential to the determination of the centre of gravity of a system when there is such a point. Some of the more remarkable propositions, which are thus established are as follows :—

If the action of terrestrial or other gravity on a rigid body is reducible to a single force in a line passing always through one point fixed relatively to the body, whatever be its position relatively to the earth or other attracting mass, that point is called its *centre of gravity*, and the body is called a *centrobaric body.*

If a body is centrobaric relatively to any one attracting mass, it is centrobaric relatively to every other; and it attracts all matter external to itself as if its own mass were collected in its centre of gravity.

The centre of gravity of a centrobaric body necessarily lies in its interior; or, in other words, can only be reached from external space by a path cutting through some of its mass.

No centrobaric body can consist of parts isolated from one another, each in space external to all; in other words, the outer boundary of every centrobaric body is a single closed surface.

A given quantity of matter may be distributed in one way, but in only one way, over any given closed surface, so as to constitute a centrobaric body with its centre of gravity at any given point within it.

Matter may be distributed in an infinite number of ways throughout a given closed space, to constitute a centrobaric body with its centre of gravity at any given point within it.

The moments of inertia of a centrobaric body are equal round all axes through its centre of inertia. In other words, all these axes are principal axes, and the body is kinetically symmetrical round its centre of inertia.

The following Gentleman was elected a Fellow of the Society:—

Professor ROBERT DYCE, M.D., Aberdeen.

The following Donations to the Library were announced:—

Insanity and Crime: a Medico-Legal Commentary on the Case of George Victor Townley. By the Editors of the Journal of Mental Science. 8vo.—*From the Authors.*

Pinetum Britannicum. Part V. *Picea Apollinis; Pinus Jeffreyii.* Folio.—*From Charles Lawson, Esq.*

Royal Geographical Society's Proceedings. Vol. VIII. No. 1 8vo.—*From the Society.*

Monthly Return of the Births, Deaths, and Marriages, registered in the Eight Principal Towns of Scotland. January 1864. (With Supplement for 1863.) 8vo.—*From the Registrar General.*

Journal of the Asiatic Society. Nos. 3 and 4. 1863. 8vo—*From the Society.*

Quarterly Journal of the Geological Society. No. 77. 8vo.—*From the Society.*

Verhandlungen der schweizerischen naturforschenden Gesellschaft bei ihren Versammlung zu Luzern den 23, 24, u. 25. Sept. 1862. 8vo.—*From the Society.*

Quarterly Return of the Births, Deaths, and Marriages, registered in the Divisions, Counties, and Districts of Scotland. No. 36. (With Supplement.) 8vo.—*From the Registrar General.*

Proceedings of the Royal Horticultural Society. Vol. IV. Nos. 2–4. (With Index to Vol. III.) 8vo.—*From the Society.*

Proceedings of the Literary and Philosophical Society of Liverpool. No. 17. 8vo.—*From the Society.*

Journal of Agriculture. March 1864. 8vo.—*From the Highland and Agricultural Society.*

American Journal of Arts and Sciences. Jan. 1864. 8vo.—*From the Conductors.*

Proceedings of the Royal Society. No. 61. 8vo.—*From the Society.*

Magnetical and Meteorological Observations made at the Government Observatory, Bombay. 1861. 4to.—*From the Government.*

Abhandlungen herausgegeben von der senckenbergischen naturforschenden Geselschaft. Band V. Heft. 1. 4to.—*From the Society.*

Sitzungsberichte der königl. bayer. Akademie der Wissenschaften zu München. 1863. II. Heft. III. 8vo.—*From the Akademie.*

Budapesti Szemle, Szerkeszti és Riadja Csengery Antal. XLI—LVII Füzet. 8vo.—*From the Society.*

Magyar Akademiai Értesítō. II. and III. 1-2. 8vo.—*From the Academy.*

Mathematikai s Természettudományi Közlemények Vonatkozólag a Hazai Viszonyokra. II. 8vo.—*From the same.*

De Finibus ultimis Nervorum Muscularium. Autore Th. Margó, Professore Universitatis Pestanæ. 4to.—*From the same.*

Phænomena continentalis elevationis et depressionis partis Europæ inter Orientem et Meridiem sitæ in Epocha Geologis "deluviali" dicta. Autore Josepho Szabó, Professore Universitatis Pestanæ. 4to.—*From the same.*

Monday, 21st March 1864.

The Hon. Lord NEAVES, Vice-President, in the Chair.

The Secretary announced that the Council had awarded the Keith prize for the biennial period 1861-63, to Professor William Thomson, of Glasgow, for his communication entitled "On some Kinematical and Dynamical Theorems."

The following Communications were read:—

1. On the Anatomy and Physiology of the Mitral, Tricuspid, and Semilunar Valves in Mammals, Birds, Reptiles, and Fishes. By James B. Pettigrew, M.D., Edinburgh; Assistant in the Museum of the Royal College of Surgeons of England. Communicated by Wm. Turner, M.B.

The writer begins his communication by giving a brief description of the structures in which valves are found, and takes occasion to comment on the nature and properties of the veins and arteries, and on the arrangement of the muscular fibres in the ventricles, as these necessarily influence, to a greater or less extent, the action of the valves. He also adverts at some length to the shape of the venous, arterial, and auriculo-ventricular orifices, and to the fibro-cartilaginous rings by which the latter are surrounded; as well as to the dilatations or sinuses which are found behind or to the outside of the segments constituting the semilunar valves in the veins and arteries, and to the shape of the ventricular cavities, which, as he points out, bear an important relation to the valves, inasmuch as they determine the direction in which the blood acts upon them; precise information on these points being, according to the author, indispensable to a just appreciation of the subject under investigation.

The object of the Memoir is to prove that in the valves, as in other structures where modifications occur, we rise from the simple to the more complex; in other words, that the valves form a differentiated and gradually ascending series.

In the veins, *e. g.*, the valve consists of a doubling of the delicate membrane lining the vessel, containing some fibro-cellular and elastic tissue, the distribution of which in the horse can be readily made out. The segments of the valve are semi-transparent, and, as a rule, semilunar in shape. They are further placed obliquely, one only being present in the smallest veins, two in the middle-sized ones, and three in the largest. Behind each segment there is a dilatation or bulging of the vessel, which projects nearly as far in an outward direction as the segment extends inwardly, and gives to the latter the requisite degree of curvature. As the dilatation referred to enables the lining membrane which forms the segments to project boldly, and almost transversely, into the vein, it follows that the free margins of the segments run for a certain distance in a nearly parallel direction, the parallelism being maintained by the presence of a fibrous structure,* which projects from the wall of the vessel, and not only supports the free margins, but carries them fairly away from the sides into the interior. This relation of the dilatations to the segments, and of the segments to each other, has not hitherto been recognised, and shows that the union between the latter is not one of the margins simply, as has been supposed, but of the margins and a considerable portion of the sides; the sides of the segments, on account of the angle at which they are set with reference to each other, being more and more involved in proportion to the amount of pressure which is brought to bear upon them. The object of the dilatations is evidently threefold—1*st* (as has been stated), To allow the free margin of the segments to project into the interior, and maintain a position with reference to each other which necessitates their coming rapidly and naturally together; 2*dly*, To increase the space occupied by the blood, and thereby extend the area, and the amount of pressure to which the segments are subjected; and, 3*dly* (which is the most important), To determine the direction in which the blood shall act, that being from above downwards, and from without inwards. The effect of this arrangement on the segments is very obvious; these, when two exist, being forced together in the mesial plane of the vessel, where they mutually support each other; and when three are present, in the axis of the vessel. In the latter case, the segments (from the more or less conical shape assumed by their free margins while in action, and from the fact of their apices being free to move) rotate to a greater or less extent, while in the act of fixing or closing—the closure being effected by a combined wedge and spiral movement.† In the veins the closing of the valves may be regarded as purely mechanical, this being due not to any active power residing in the segments, but to the pressure exerted upon them by the blood, and to the vital contractility residing in the

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* The structure adverted to is very well seen in the arterial semilunar valves.

† The spiral wedging movement is especially distinct in the segments of the arterial semilunar valves.

veins, which tends to urge the segments together, while it assists the onward flow of the blood. In the arteries, as, *e. g.*, at the origin of the pulmonary artery and aorta, the segments of the valve are of a semilunar shape, and always three in number. They consist of a doubling of the lining membrane, and contain within their fold certain tendinous bands which run in well-marked directions; the structure closely resembling that met with in the auriculo-ventricular or highest form of valve.* The cusps, moreover, are semi-opaque, stronger, and more ample than in the veins.

In the arteries the dilatations behind the segments, commonly called the Sinuses of Valsalva, are very large, and, as the author has ascertained, curve towards each other in such a manner as ensures that the blood will act upon the segments only in certain directions: in fact, that they will be urged by it towards each other, and fixed by a distinctly spiral wedging movement; the margins of the segments flattening themselves against each other whilst in action, and forming an inverted tripartite dome, the strength of which is limited only by the strength of the materials used in its construction. The action of the arterial semilunar valves may also be considered mechanical, the inconvenience which might be supposed to result from an excess of vital contractility in the arteries and ventricles, between which the valves are situated, being counteracted by the existence at this point of comparatively unyielding fibro-cartilaginous rings. Strongly contrasting with the arterial semilunar valves in mammals whose area of activity is, so to speak, circumscribed, are the semilunar valves situated within the bulbus arteriosus of fishes, which is an actively contracting structure. In these cases the segments, as a rule, are more numerous than in the veins and arteries, and arranged in tiers, so that the blood which is not caught by one tier falls into and is supported by the next. The number of segments, moreover, obviates the evil effects which might accrue from the displacement to which they are subjected by the contraction of the bulb.

In the semilunar valves of fishes we find the first trace of what may be regarded as chordæ tendineæ, in the shape of tendinous

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* The arrangement of the tendinous bands in the semilunar and auriculo-ventricular valves, and the structure generally, is described at length.

bands which run between the segments and afford a certain degree of support. The action of the valves in fishes is partly mechanical and partly vital, for we must regard the contraction of the bulb as contributing to the closure. In the auriculo-ventricular valves of mammals, birds, reptiles, and fishes, we have a great variety. These valves are characterised by the presence of tendinous chords (chordæ tendineæ) which connect them with actively contracting muscular structures; as, the interior of the ventricles or the structures arising therefrom—viz., the carneæ columnæ and musculi papillares. They therefore differ from the semilunar valves proper. In the auriculo-ventricular valves there is also gradation. In some instances, *e.g.*, there is only one semilunar flap or segment; in a second there are two flaps or segments, so arranged *that their long diameters correspond to the direction of the muscular fibres with which they are connected directly.* In a third, the two segments are attached to the interior of the ventricle *by rudimentary chordæ tendineæ.* In a fourth, two accessory or smaller segments are added to the two principal ones, *the whole being attached by well-developed chordæ tendineæ to rudimentary musculi papillares.* In a fifth, which is the most perfect form of valve as it exists in man, and in the higher mammalia, the segments are from four to six in number, *most exquisitely and symmetrically formed, and attached by minutely graduated chordæ tendineæ to highly developed carneæ columnæ and musculi papillares;* the latter being distinctly spiral.

The action of the auriculo ventricular valves is varied, and depends on the circulation, on the configuration of the ventricles, and the shape of the ventricular cavities, which adapt and mould the blood, and cause it to act in a definite or given direction. In the Fish, the ventricle and the ventricular cavity are pyramidal in shape, the ventricular fibres being so arranged that the organ contracts and dilates very much as one would shut and open the hand. As, moreover, the circulation is languid, the segments of the valve, when two exist, are forced towards each other, by the contraction of the ventricle and by the blood, in a manner analogous to that by which the segments of the bisemilunar venous valves are approximated, by the retrogressive movements of the slowly advancing venous blood, assisted to a slight extent by the vital contractility of the vessels. In the Reptile, where the circulation is also languid or slow, the shape of the ventricle or ventricles,* and of the ventricular cavity or cavities, is conical; the cone being slightly twisted upon itself. As the muscular fibres forming the ventricle pursue a more or less spiral course, and this arrangement extends to the valves, their action may be aptly compared to that which obtains in the valves of the largest veins, and in those of the arteries. It is, however, in the auriculo-ventricular valves of the Bird and Mammal that the spiral action of the segments becomes most conspicuous; the nature of the action being unavoidably determined by the spiral arrangement of the muscular fibres forming the ventricles, and by the spiral nature of the musculi papillares and ventricular cavities. As the two spiral musculi papillares project into the spiral ventricular cavities, it follows that between them there exist two spiral grooves or depressions, and in these the blood is arranged, on its entrance into the ventricles, in two spiral columns; that fluid, towards the end of the diastole and the beginning of the systole, advancing on the segments of the auriculo ventricular valves in spiral waves, from below upwards, and wedging and screwing them into each other in an upward spiral direction; the musculi papillares, by contracting, dragging the segments towards the end of the systole, by means of the chordæ tendineæ, in an opposite or downward direction, to form a spiral dependent cone, the apex of which points to the apex of the heart. The action of the auriculo ventricular valves is partly mechanical and partly vital, the musculi papillares and analogous structures exercising, through the chordæ tendineæ, at one time a restraining influence to prevent retroversion and regurgitation, at another wielding a direct power for approximating and applying them accurately to each other. The paper, which is based on an extensive series of dissections, is illustrated by upwards of 50 photographs and drawings, showing the structure, relations, and action of the valves.

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* In some cases, as is well known, the almost perfect septum divides the ventricle into two.

2. On some points in the Metrology of the Great Pyramid. By Professor C. Piazzi Smyth, Astronomer Royal for Scotland.

This paper was an attempt to submit to a severe and searching examination, the very new and apparently important ideas contained in the work, published four years ago by Mr John Taylor of London, and entitled "The Great Pyramid; why was it Built?" To this end, the original authorities for measures of the Pyramid, had been extensively referred to, from Professor John Greaves in the 17th century, down to Colonel Howard Vyse and Dr Lepsius in the 19th; and their various and sometimes conflicting numerical statements had been computed with all due attention to scientific accuracy, as well as every endeavour to eliminate both personal and other sources of error in the observations.

The result of this proceeding has been most eminently favourable to all the more important and cogent of Mr Taylor's conclusions; not only as to the probability of a common origin in prehistoric times for the *hereditary* weights and measures of all nations, but as to there having been something more than mere human intelligence concerned in their establishment; and also, to the Great Pyramid, besides having been earlier in date, being, in all the spirit of its construction, completely separated from all the other Pyramids, as well as from everything usually thought typical of Egypt, or peculiar to the Pharaonic, or any other, dynasty of the Egyptians; and entirely devoid, in its finished parts, of even the remotest and most distant allusion to any form of idolatrous worship ever practised by any nation of antiquity whatever.

The calculations were prepared in full, and an astonishing series of coincidences upon coincidences given, both as to the relations truly connecting the dimensions of the Great Pyramid, with the latest geodesically-determined numbers for the size and figure of the earth; to the original founding of the standard of that Pyramid's linear measure on the one and only absolutely correct scientific reference which the earth contains—viz., the axis of rotation; to the connecting of its ancient measures of capacity and weight with the weight, or what goes, with the size, to make the weight, of the earth; and to the identification of the very convenient divisor typical to the Pyramid, even in the temperature secured for its standards.

There then followed a comparison of the ancient Pyramid-measures, in length, surface, capacity, and weight, with the English *hereditary* measures still in existence among us; and it showed, first, the extreme closeness of the two systems, especially in the smaller units employed chiefly by the working and poorer classes of our country; and then, the remarkable convenience for scientific purposes and questions of the present day, of the larger Pyramid-units; of which, not the least striking example is, that the large Ordnance map of Great Britain recently commenced, and now being rapidly pushed on by the Government, on a scale of $\frac{1}{2500}$th of nature, the largest, best, and most expensive map of our country which has ever yet been attempted,—but which does not fit into the present British measures at all, requiring long and annoying fractions both for the mile and the acre, to the plague of every British-born working-man who uses it,—fits in *precisely* to the ancient Pyramid measures of 4300 years ago, and represents their mile and their acre *true*, in inches, without the smallest fraction of an inch left over or under.

3. The Decimal Problem Solved. By James Alexander, Esq. Communicated by E. Sang, Esq.

The author having had experience of the simple duodecimal monetary system of Portugal, which is accumulated by multiplication from a decimal minimum unit, proposes to apply the same principle to the British coinage. Having also discovered, while in Canada, that the difference betwixt the British coinage and a decimal position or character, is 20 per cent. over all, he proposes to overcome this by altering the value of the present farthing, by this rate, making it the fifth, instead of the fourth of the penny—the penny of 5 farthings would then become the duodecimal multiplier of all the coins, making the shilling 60 farthings instead of 48, and the pound 1200 instead of 960, and each intermediate coin would be represented by one of the digits and one or two ciphers, while the pound, being represented by 12, the highest single divisor or multiplier, could be kept out as the maximum unit. This would introduce the decimal principle into the coinage, without sacrificing either the pound or the penny, which it has hitherto been declared impossible to do, while it would completely revolutionise our arithmetic, and bring our money into accord with the whole world. By this slight alteration, making no disturbance in the names or values of the coinage—affecting nobody injuriously, but everybody beneficially—a change of notation from the present £ *s.* *d.* is made imperative upon no one who prefers it; but the great mass of traders may at once keep their accounts in pounds and 1200ths, while the larger merchants and bankers may keep them in pounds and 120ths, greatly to the diminution of figuring and arithmetical labour, while the conversion of the decimal into *s.* and *d.* is the simplest mental operation.

4. On the Elevation of the Earth's Surface Temperature produced by Underground Heat. By Professor W. Thomson, F.R.S., F.R.S.E.

Peclet found, by his own experiments, that a body with any common unpolished non-metallic surface, kept by heat from within at 1° higher temperature than that of the air and other objects round it, loses heat from each square metre of surface at the rate of about nine kilogramme-water thermal units per hour, or, which is the same, $\frac{1}{4000}$th of a gramme-water unit from each square centimetre per second. The mean conductivity of the three Edinburgh strata, in which Principal Forbes's underground thermometers were placed, is $2\frac{3}{4}$ grain-water units per second per square foot per 1° per foot rate of variation of temperature, as I have shown previously.*

That of the Greenwich stratum is 2·6, in terms of the same units, according to Professor Everett's recent reductions; and that of certain strata (clay and sand) in Sweden is 1·64, according to Ångström (Poggendorff's *Annalen*, last volume of 1861). The mean of these three numbers is 2·33, which, reduced to the unit of conductivity founded on the gramme-water-second-centimetre units, is ·005. Taking this, therefore, as an average conductivity in the earth's upper crust, we find that if the temperature increased downwards at the rate of 1° per 20 centimetres, the quantity of heat lost by conduction outwards would be $\frac{1}{4000}$th; and therefore, according to Peclet's result, this would keep the surface just 1° warmer than it would be if there were no conduction of heat from within. Hence, to warm the surface to 10° Fahr. above what it would be if there were no conduction from within, the rate of rise of temperature must be 1° Fahr. per 2 centimetres, or ·0656 of a foot (which would probably destroy the roots of any large tree or plant), but at all events could not, as I have shown,* be the real condition of the earth at any time later than about 180 years after even a greater heating (7000° Fahr.) of the whole globe than the greatest we can suppose it at all probable the earth ever experienced. Hence it is certain that the climate can never have been sensibly influenced from the earliest "geological" era by underground heat. This conclusion was stated in § 17 of the paper already referred to "On the Secular Cooling of the Earth," as rendered certain by a rough general knowledge of the circumstances, without any approach to an accurate estimate of the absolute amount of radiation.

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* Trans. R.S.E., April 1860, "On the Reduction of Observations of Underground Temperature," § 42.

We now see, farther, that the present rate of underground rise of temperature, estimated at 1° Fahr. per 50 feet, is only $\frac{1}{75}$th of that which is required to warm the surface by 1°. Hence the surface is only about $\frac{1}{75}$th of 1° Fahr. warmer at present than it would be if there were no supply of heat from within.

The following Donations to the Library were announced:—

The Canadian Journal of Industry, Science and Art. No. 49, 8vo. —*From the Canadian Institute.*

De l'Origine des lacs Suisses, par M. B. Studer. 8vo.—*From the Author.*

Proceedings of the Royal Geographical Society. Vol. VIII. No. 2. 8vo.—*From the Society.*

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* Trans. R.S.E., April 1862, "On the Secular Cooling of the Earth," § 18.

Journal of the Chemical Society. March 1864. 8vo.—*From the Society.*

Monthly Return of the Births, Deaths, and Marriages registered in the Eight Principal Towns of Scotland, &c. February 1864. 8vo.—*From the Registrar-General.*

Monthly Notices of the Royal Astronomical Society. Vol. XXIV. No. 4. 8vo.—*From the Society.*

Bulletin de la Société de Géographie. Cinquième Sériè. Tome VI. 8vo.—*From the Society.*

Sitzungsberichte der könig̈. bayer. Akademie der Wissenschaften zu München. II. Heft V. 8vo.—*From the Academy.*

Mémoires de l'Académie des Sciences de l'Institut Impérial de France. Tome XXVI. 4to.—*From the Academy.*

Extraits de Géologie pour l'année 1861. 8vo.—*From M. Delesse.*

Proceedings of the Royal Horticultural Society. Vol. IV. March 1864. 8vo.—*From the Society.*

Monday, 4th April 1864.

Professor INNES, Vice-President, in the Chair.

The following communications were read:—

1. On the Celtic Languages in their Relation to each other, and to the Teutonic Dialects. By W. F. Skene, Esq.

(*This paper was given at the request of the Council.*)

Mr Skene in his paper gave a sketch of the various views which had been entertained regarding the Celtic languages, and advocated the opinion that these languages belonged properly to the Indo-European class. He gave an outline of the distinctive peculiarities of the two branches of the Celtic languages, the Welsh and the Gaelic, and discussed the leading theories regarding the lost Pictish language, only five words of which were now extant. Mr Skene had, as the result of his investigations, arrived at the conclusion that the Pictish language occupied a place between the Welsh and the Gaelic—that it was a Gaelic dialect partaking largely of Welsh forms.

2. On the Protection of Vegetation from Destructive Cold every night. By Professor W. Thomson.

The effect of dew in protecting vegetation every clear still night of summer was long ago pointed out by Dr Wells; the correctness and acuteness of whose views on this subject have been generally recognised. The hypothesis recently put forth by Dr Tyndall, that absorption of radiant heat by aqueous vapour in the atmosphere is an effective defence against destructive degrees of cold, and the ready acceptance yielded to it by some of our highest authorities in the popular promulgation of the truths of science, seem to render it necessary to recall attention to Dr Wells' admirable work. In the first place, when Dr Tyndall announces, as a result of his experiments on radiant heat, that, "It is perfectly certain that more than ten per cent. of the terrestrial radiation from the soil of England is stopped within 10 feet of the surface of the soil," by the absorption it suffers from aqueous vapour; it must be remarked that this absorption cannot go on at the same rate through any great thickness of air. For at the same rate half the radiant heat would be absorbed in 70 feet; $\frac{3}{4}$ in 140 feet; $\frac{7}{8}$ in 210 feet, and so on, which is inconsistent with known facts; as, for instance, the influence of clouds on terrestrial radiation. Hence the quality of rays which passes through the lowest 10 feet of air suffers less than ten per cent. of absorption in the next 10 feet; and it is quite certain that after passing through several times 10 feet of air, the radiant heat must, by having been deprived of the part of it liable to absorption by aqueous vapour, be in a condition in which not one per cent. is absorbed from it in its passage through 10 feet of clear air. Whatever influence true vapour of water really does exercise in checking, by its absorption, the loss of heat by radiation from the earth's surface, it is, even in the most humid conditions of optically clear atmosphere, insufficient to prevent heavy dews; far less than the latent heat of which, taken from the blades of grass, or other finer parts of plants, would leave them destroyed by frost.

In point of fact, therefore, heat *actually is radiated* away into very high terrestrial atmosphere and distant interstellar air or æther, from the upper and finer parts of living plants, in so great amount every clear night of summer, that destruction by frost could not be delayed for many hours after sunset without a compensating supply of heat from some extraneous source. This source, on windy nights, is the thermal capacity of the air whirled about, up and down, and among the stems and leaves of the plants. On still nights it is the latent heat of the vapour condensed into dew. This vapour is taken chiefly from the air engaged among the stems and leaves, which, in the case at least of fine grass, is all *nearly* at the same temperature as the leaves; the temperature of the surface of these being of course rigorously the same as that of the air in contact. Thus the temperature of the leaves can never go *below* the *dew-point* of the air touching them, and any cooling which they experience *after* dew begins to deposit upon them is only equal to the lowering of the *dew-point*, occasioned by the amount of drying experienced by the air in consequence of the condensation of vapour out of it.

Clouds, as remarked first by Prévost, being practically opaque, prevent the surface of the earth from *tending* by radiation to a lower temperature than their own, which, unless they are very high, is generally not much colder than the dew-point of the lower air, but is at all events in general sufficiently warm to prevent the finest blades of grass from acquiring any very sensible dew, or to allow the general temperature of grass and the air engaged among it, even on the stillest night, to sink as low as the dew-point. Thus, either clouds, by their counter-radiation, or wind, by mixing a comparatively thick stratum of air with that next the earth, keep the grass and delicate parts of other plants from sinking to the dew-point; or, when there is not enough of clouds and wind to afford this degree of protection, dew begins to form, and by preventing the temperature of any leaf or flower from sinking below the dew-point, saves them all from destruction, unless, as when hoar-frost appears, the dew-point itself is below the freezing point.

The following Donations to the Library were laid on the Table :—

Catalogue of the Advocates Library. Part I. A—AZZ. 4to.—*From the Librarian.*

Census of Scotland. Population Tables and Report 1861. Vols. I. and II., folio.—*From the Registrar-General.*

Annual Detailed Reports of the Births, Deaths, and Marriages in Scotland. Vols. I.–V. 8vo.—*From the same.*

The Battle of the Standards, the Ancient of four thousand years against the Modern of the last fifty years—the less perfect of the two. By John Taylor, Esq. 8vo.—*From the Author.*

Dana on the Classification of Animals on the Principle of Cephalization, &c. 8vo.—*From the Author.*

Journal of the Statistical Society. March 1864. 8vo.—*From the Society.*

Memoirs of the Geological Survey of India. **2** . 6 . **3** . 1. 4to.—*From Dr Oldham.*

Turgan, Les Grandes Usines de France, Pépinières d'André Leroy à Angers. 8vo.—*From the Author.*

Premium Report on the Progress of the more recently introduced *Coniferæ.* By Robert Hutchison, Esq. 8vo.—*From the Author.*

Reale Instituto Lombardo di Scienze e Lettere. Rendiconti : Classe di Scienze Matematiche e Naturali. Volume I. Fasc. I. e II. Gennaio—Febbraio. 8vo.—*From the Institute.*

Geological Survey of Canada, 1863. 8vo.—*From Lord Monck.*

Proceedings of the British Meteorological Society. Vol. II. No. 10. 8vo.—*From the Society.*

Ricerche Storiche sulla legatura delle Vene e delle Arterie da Celso a Dionis per Giuseppe Longo da Casarano. 8vo.—*From the Author.*

The Shorter Catechism with proofs, in Hebrew and Syriac. By the Rev. H. S. M'Kee, D.D., LL.D. Killimane. 24mo. 1864.—*From the Author.*

Monday, 18*th April* 1864.

Sir DAVID BREWSTER, Vice-President, in the Chair.

In presenting the Keith Medal to Professor **William Thomson** of the University of Glasgow, Sir **David Brewster** made the following remarks:—

It is now nearly half a century since the late Mr Keith, of Dunottar, intimated to me his intention to bequeath L.1000 for the promotion of Science in Scotland, and to appoint me one of the Trustees for the administration of the Fund. I cheerfully accepted the trust, and pointed out to Mr Keith the manner in which the fund might be most advantageously employed.

In conformity with this plan, the sum of L.600 was presented to the Royal Society, "to form a biennial prize for the most important discovery in Science made in any part of the world, but communicated by its author to the Society, and published in its Transactions." The rest of the Fund, which had increased considerably, was subsequently presented to the two Societies in Edinburgh, which had been instituted for the promotion of the Fine and the Useful arts.

Agreeably to the regulations adopted by the Council, the Keith Prize for 1862 and 1863 fell to be allotted to the most important discoveries in Physical Science, made during these two years. In deciding upon its adjudication, the task of the Council was as easy as it was agreeable. Professor William Thomson, who was elected a Fellow of the Society in 1847, had, during the last seventeen years, communicated many valuable papers to the Society, which added greatly to the value of its Transactions. These papers, and others elsewhere published, relate principally to the Theories of Electricity, Magnetism, and Heat, and evince a genius for the mathematical treatment of physical questions, which has not been surpassed, if equalled, by that of any living philosopher.

In studying the mathematical theory of Electricity, he greatly extended the general theorems demonstrated by our distinguished countryman, Mr Green, and was led to the principle of "*Electrical Images*," by which he was enabled to solve many problems respecting the distribution of electricity on conductors, which had been regarded as almost hopeless by the most eminent mathematicians in Europe.

In his researches on Thermo-dynamics, Professor Thomson has been equally successful. In his paper "On the Dynamical Theory of Heat," published in our Transactions for 1851, he has applied the fundamental propositions of the theory to bodies of all kinds, and has deduced many curious and important results regarding the specific heats of bodies, which have been completely verified by the accurate experiments of Mr Joule.

No less important are Professor Thomson's researches on Solar Heat, contained in his remarkable paper "On the Mechanical Energy of the Solar System;" his researches on the Conservation of Energy as applied to organic as well as inorganic processes; and his fine theory of the Dissipation of Energy, as given in his paper "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy." To these we may add his complete Theory of Diamagnetic Action, and his investigations relative to the Secular Cooling of our Globe, and the influence of Internal Heat upon the temperature of its surface.

The value of labours like these could not escape the notice of the Council of this Society, and they would have entitled their author to the Keith Prize, had they not been presented to the Society when the Prize was devoted to other branches of Science. It is not, therefore, for these researches and discoveries that the Keith Medal has been awarded to Professor Thomson, but for the very interesting and important discovery, in Abstract Dynamics, which he has communicated to the Society during the biennial period appropriated to Physical Science.

By the previous researches of Euler, Lagrange, Delaunay, and Bertrand, it had been established that when any system of bodies, connected by any invariable kinematic relations, is struck with impulses of any kind, the kinetic energy thus developed is a maximum. This remarkable principle is of very great use in the investigation of certain complex dynamical problems, but in many important cases it is inapplicable. In the motion, for example, of an incompressible liquid, contained in a vessel whose form is suddenly altered, it is not the impulse on the liquid which is given, but definite kinematic relations are laid down to which the impulsive motion must be subject. In such a case, Professor Thomson has shown that the kinetic energy which has been thus generated is a minimum; and it is for this theorem that the Prize has been awarded,—a theorem which not only affords a direct and simple mode of dealing with questions of more than ordinary difficulty, but which is itself an extremely singular and valuable property of Motion.

The researches of Professor Thomson, to which we have referred, are of too recondite a nature to be generally appreciated. The results of abstract Science have ever failed to excite the notice, or call forth the gratitude, of the State. The question of practical use—the *cui bono* of ignorance—is put every day by educated men in high and responsible positions, to whom are confided the mightiest interests of the nation. History and experience might have taught them, had they been capable of learning, that abstract Science is the life and soul of Industry, and that its successful cultivators are the true benefactors, not of their country only, but of their species. The discoveries of our eminent colleague, and even the very abstract property of motion, to which we have attached so special a value, must, like all similar revelations, find invaluable applications to the material interests of society.

I should be doing injustice, however, to Professor Thomson were I to characterise his researches as wholly abstract and transcendental. The important conclusions which he obtained from the Theory of Induction in submarine Telegraphs, have found a valuable practical application in the Patent Instruments for reading and receiving messages, which he so successfully employed on the submarine cable across the Atlantic; and when that great work is completed, his name will be associated with the noblest gift that Science ever offered to Civilisation. By his delicate Electrometers, his Electric Spark Recorder, and his Marine and Land Reflection Galvanometers, he has provided the world of thought with the finest instruments of observation and research; and the world of action with the means of carrying the messages of commerce and civilisation which have yet to cross the uncabled oceans that separate the families of the earth.

Entertaining these views of the services which my distinguished friend has performed to Science and the Arts, I have much pleasure in expressing the same sentiments on the part of the Society, and in being their organ in delivering to him the Prize which he has so justly merited.

In discharging this duty, I am proud to think, and I am sure that all here will participate in the sentiment, that Scottish Science has such a representative in the University of the West, while, in our own, it has one of kindred genius and power.

Professor Thomson, I now beg to present to you the Keith Medal, and to congratulate you on the honour which you have so justly deserved.

1. On Vital Agency with reference to the Correlation of Forces. By William Seller, M.D., F.R.S.E., Fellow of the Royal College of Physicians of Edinburgh.

In this paper there is presented a series of characteristic examples of vital agency, followed by some considerations bearing on the extent to which, in a physiological point of view, it can be admitted that the physical forces, under exchangeable forms, are concerned in organic phenomena. At the same time, one of the particular objects of inquiry is, whether there be ground for the belief that vital agency, being not a force but a directive principle, may accomplish all the parts assigned to it, solely by means of the physical forces variously modified through organic structure.

It is laid down among the preliminaries, that no facts hitherto discovered bring the origin of organic life within the pale of inductive inquiry; that the transmutation of species by natural selection, though a brilliant conception, holds as yet no foundation of an inductive character; that it remains therefore a fundamental fact in physiology, admitting of no explanation, that all species have their commencement in original parentage; and further, that it is thence correct in principle to maintain that all the multiplied phenomena in the life of a species, of whatever extent, derive their source from the potentiality of the reproductive cells in the primordial parent or parents of that species.

It is also taken for granted that such a directive principle, as that already indicated, must act through the organism and the organic fluids which take their first origin in this manner.

The character of nutrition throughout organic nature is then dwelt on, with the purpose of showing that it is everywhere under the control of more or less perfect organic (physiological) atoms; that even reproduction of species is nutrition of a more special kind, the organic atoms supplied by each sex being thrown off only when the individual or individuals whence they are derived have attained a very perfect maturity.

The maintenance of heat in organic bodies, and particularly in animals of perfect type, is next pointed out as probably indicating the true nature of the connection between vital agency and the physical forces, since in every case the higher temperature has a purely physical source, while it is nevertheless under the control of vital agency.

A rapid view is next taken of acts of relation in the animal world, as more immediately proving the necessity for a directive principle to explain, in any degree, the reference of such acts to physical forces.

Finally, the interference of a directive principle is represented as essential to show why vital agency at last puts a limit to its own operations, even when the material supplied is in abundance, first building up the springs of growth and limiting the size of each individual over which it presides, in accordance with the rule of the species; next building up the springs of reproduction earlier in the one sex than in the other; and, lastly, building up the springs of life itself, and terminating the career of its own individual existence in a manner even less intelligible to finite understandings than any of the most wonderful of its acts of workmanship in the greatest vigour of its power.

A few passages are then devoted to the exhibition of views taken by Carpenter, Leconte, also by Thomson and Tait, in speaking of organic phenomena with reference to the correlation of physical forces or to the conservation of energy.

Affinity and aggregation are next reviewed at some length, with the view of showing that there is no essential difference between affinity in inorganic nature and that affinity which builds up and repairs organic structures when the agency of a directive principle is admitted; that that directive principle operates through organic structure and the protoplasma in vegetable bodies, both of which owe their origin in unvaried succession to the original parents of the species; that all organic structure at present in existence may be traced back to the state of vegetable protoplasma; that the protoplasma in the cells of the green parts of plants becomes augmented by the assimilation of carbon and hydrogen in particular, derived from the carbonic acid and water of the atmosphere, and that this operation is purely chemical, aided by the merely physical agency of light; that the presence of water in so large proportion throughout the living solids, pervading their atoms and interstices like an almost incompressible atmosphere, accounts for many of the peculiarities of organic affinity, and among others for the apparent antagonism between the attraction which holds together the living solids and that which, on the failure of vital action, so quickly reduces them to the state of purely mineral matter—the case being exactly parallel to that of water containing salts in dilute solution, the concentration of which is followed by the precipitation of such insoluble salts as can arise out of the constituents of the salts originally dissolved in the fluid—finally, that the heat which attends the exercise of such affinities as produce water and carbonic acid is due to the rapid movement of the strongly attracted atoms, that being to a far greater degree than in the exercise of the organic affinities.

It is next considered how far electricity is to be regarded as concerned under a directive principle in the phenomena of life, and particularly in the exertion of animal power.

The facts ascertained with respect to electrical fishes are pointed to as sufficient proof, that animals nourished and constituted like vertebrated animals can generate a great amount of electrical force, the peculiarities of the electrical fishes being in fact inconsiderable, when compared with the effects produced. It is further noticed, that the general result of the many experiments made by recent electro-physiologists is to show, that during life, hardly any two parts of the animal body are in electric equilibrium. From all which, and many other facts, there appears to be nothing improbable in the idea that electricity is an active agent in the phenomena of life, more especially in those of the nervous apparatus and the muscular system. Moreover, from what appears to be the fact, that electricity is developed along with heat in the eremacausis or organic decomposition under the influence of oxygen attendant on the occurrence of vital phenomena, while the living body exists in such a non-conducting medium as air; the unsolved question already presents itself—What becomes of that electricity? At the same time, however, it is acknowledged, that as respects physiology, electricity is full of promise rather than of actual results. It is shown that the numerous, and, in some respects, contradictory experiments of Du Bois Reymond and Matteucci, prove that electricity exists in living bodies, but do not conclusively indicate what it does or can perform.

The question of most immediate interest at present in connection with the correlation of physical forces, is whether electricity or heat, or partly the one, partly the other, be the representative of animal power in muscular exertion.

If vital agency be assumed to be not a force—that is, not an immediate source of motion—but only a directive principle analogous to man's intelligence when it compels the properties of bodies and the laws of nature to minister to the fulfilment of ends suggested to him by his appetites, desires, and capacities of enjoyment, then, to avoid the alternative of representing force as springing out of nothing, animal force, as manifested in muscular action, must be considered as metamorphosed from affinity, electricity, or heat. In continuance, a muscle can be conceived to be so built up by affinities, as that it shall, under certain conditions, shorten itself and produce a mechanical effect; but such an effect can take place in this manner only once—to contract again and again in quick succession by the expenditure of affinity, it would require to be rebuilt after each contraction, which seems next to impossible. A muscle, therefore, after one contraction, is in the same predicament as a weight that has fallen from a height to the ground, which, to produce the same effect a second time, must be raised to the same elevation as before. Is it, then, electricity, or heat, or partly the one partly the other, which produces the renovating effect on the muscle? One of the illustrations on this point is the following:—Some physiologists regard a muscle as being in its natural or spontaneous state when in complete contraction, believing, that when the body is at rest, the molecular structure of the muscle is kept by some kind of force in a species of constraint—that on the discharge of this force the muscular fibril contracts, and that the constraining force is immediately renewed. It has been supposed that the force here operative is heat, yet electricity seems better fitted for the purpose. In support of this view is cited the analogy, indicated by some authorities, between the structure of the muscular system and that of the electric organs in the gymnotus and torpedo.

If such a view, under whatever modification, be rejected, heat must be resorted to as the source of animal power.

To prove heat to be the source of animal power, it must be shown that the products of eremacausis, under forcible muscular exertion, both account for the whole temperature manifested in the meantime, and leave a surplus sufficient to be metamorphosed into the calculated equivalent of the mechanical effect produced by the entire amount of that exertion.

Chemists pronounce that the amount of heat in the combustion of hydrogen and carbon, whether slow or rapid, is exactly proportioned to the products in water and carbonic acid; nevertheless, it may be doubted if two circumstances have been taken into account in their estimate as respects the living body, viz., the disturbance of electric equilibrium in such combustion, and the reciprocal convertibility of heat and electricity, so that it may turn out that, with the same amount of material products, there may be room for a reciprocal variation in the proportions of these forces developed.

Whence, though the product in carbonic acid and water be in exact proportion to a given exertion of animal power, the question appears to be still unsolved, Whether heat, or electricity, or both jointly, represent the equivalent of mechanical force put forth in such exertions.

2. On Sun Spots, and their connection with Planetary Configurations. By Balfour Stewart, Esq. Communicated by Professor Tait.

The author was led to examine the sun pictures taken by the Kew Photoheliograph, with the view of ascertaining if any connection exists between the behaviour of sun spots and planetary configurations.

It was found, that when any portion of the sun's disc recedes by virtue of rotation from the neighbourhood of Venus, it acquires a tendency to break out into spots, and, on the other hand, when such approaches Venus, there is a tendency towards the healing up of spots. Carrington's observations were then discussed, which seem to show that, on the whole, the sun's surface is fullest of spots when Jupiter is furthest from our luminary, and freest from spots when he is nearest. This action of Jupiter is not, however, much influenced by the rotation of our luminary, perhaps because the sun's diameter is small compared with the distance of that planet. The mode of action of Venus and Jupiter may both be expressed by the following law:—" When the sun's disc, or part of it, approaches a planet, or when a planet approaches the sun's disc, there is an absence of spots, or a tendency to luminosity produced."

It was then shown that this law was sufficient to explain the phenomena of variable and of temporary stars; and, in conjunction with Professor Tait, the author suggested that it might be analogous to that in virtue of which two atoms rushing together give rise to radiant light and heat.

3. Biographical Sketch of Adam Ferguson, LL.D., F.R.S.E., Professor of Moral Philosophy in the University of Edinburgh. By John Small, Esq., M.A., Librarian to the University of Edinburgh. Communicated by Professor Fraser.

The following Donations to the Library were announced:—

Almanaque Náutico para 1865, Calculado de órden de S.M., en el Observatorio de Marina de la Ciudad de San Fernando, 8vo.—*From the San Fernando Observatory.*

Journal of the Proceedings of the Linnean Society. Vol. VII., No. 28. 8vo.—*From the Society.*

Monthly Return of the Births, Deaths, and Marriages registered in the Eight principal Towns of Scotland—March 1864. 8vo.—*From the Registrar-General.*

Journal of the Chemical Society. April 1864. 8vo.—*From the Society.*

Monthly Notices of the Royal Astronomical Society. Vol. XXIV. No. 5. 8vo.—*From the Society.*

Proceedings of the Royal Horticultural Society. Vol. IV. No. 6. 8vo.—*From the Society.*

Proceedings of the Royal Society. Vol. XIII. No. 62. 8vo.—*From the Society.*

On the Vertebroid Homologies of the Cranium in Vertebralia or Osteozoa, and the Analogous Homologies of the Annulozoa or Articulata. By William Macdonald, M.D., F.R.S.E., &c. 8vo. —*From the Author.*

Monday, 2d May 1864.

PROFESSOR LYON PLAYFAIR in the Chair.

Some of the following Communications were read:—

1. Unpublished Letter of the late Professor Dugald Stewart. Transmitted by John Small, M.A., Librarian to the University. Communicated by the Rev. Dr Stevenson.

The letter which Mr Small has sent to the Royal Society, and which, at the request of the Council, I have the honour of bringing under the notice of the meeting, was acquired by the University in 1861, at the sale of the MSS. of the late Very Rev. Principal Lee. It is by no means destitute of historical importance, though its main value for us may rest on the consideration that it was written directly from the scene of the convulsion which it describes, by Professor Dugald Stewart to the Reverend Archibald Alison. As a zealous student of political philosophy, Mr Stewart took a lively interest in the early stages of the French Revolution. He accordingly visited Paris both in 1788 and in 1789; and while there he maintained an active correspondence on the subject which engrossed him, with his friend Mr Alison. Of this correspondence five letters, unconnected with one another, have been printed in the Appendix to Professor Veitch's Memoir, which forms the first article in the tenth volume of Stewart's collected works. These five are dated, 27th August 1788, 10th May 1789, 30th May and 4th June 1789, 27th June 1789, and 17th September 1789.

The letter now brought to light by Mr Small, the most interesting of the series, so far as it is known to have been preserved, is dated 2d July 1789, and is distinctly connected with that already printed, of date 27th June. In the latter the writer says:—" My Dear Archy—I had begun a very long letter for you, which I find I cannot possibly get ready for you till next post. I must therefore content myself at present with mentioning to you, that the day before yesterday, *i.e.* 25th June, the king sent a letter to the clergy and noblesse, desiring them by every consideration of regard for his person, and of attachment to their country, to unite themselves instantly with the Tiers Etat, which they did that very day." That the document now before us is the " very long letter" thus referred to, becomes obvious from the facts, that it takes up the narrative of events at the 22d of the month, and that, towards the close, it mentions the union of the clergy and noblesse with the Tiers Etat, in nearly the precise words quoted above.

On the 17th of June, the Tiers Etat had declared themselves a " National Assembly," in consequence of which bold step, it was determined by the Court party to close the hall of the said Tiers Etat till June 23. On the 20th, proclamation to this effect was made by the Herald-at-Arms; but the " Assembly," excluded from their own hall, met in the Tennis-Court, and then took the famous Tennis-Court oath—with only one dissentient. On the 22d, no fewer than 148 of the clergy joined the Commons or Tiers Etat, now assuming to itself the unusual name and the boundless powers of " L'Assemblée Nationale." On the next day, the 23d, came the Séance Royale, according to previous proclamation; and here the letter of Mr Stewart takes up the narrative.

" My Dear Archy,—My last letter, if I recollect right, was dated on Sunday, 21st June, at which time everything here was in the utmost confusion. On Monday morning, notice was given in Paris that the *Séance Royale* would not take place till *Tuesday;* and in the meantime copies were distributed of a letter from M. Necker to M. De Crosne (who has the charge of the police in Paris), tending to quiet the apprehensions of the people about the dissolution of the States General, and assuring them that the K. was doing everything in his power to conciliate the different orders. On

Tuesday forenoon I went to Versailles, chiefly from a desire of getting the earliest intelligence of the result (for I had heard before that no strangers were to be admitted into the Assembly), and I arrived just after the time the *Séance Royale* was ended. Everybody seemed to be in the greatest consternation, for the King had gone to the *Salle, without M. Necker*, accompanied by the *Garde des Sceaux* and the other ministers. The noblesse had retired with the King, together with a great part of the clergy; but the Deputies of the people were still sitting, although the K. had given them positive orders to separate immediately, and to assemble again the day following. At that time everybody believed that M. Necker was out of office, and it was generally supposed that the Prince of Conti would be at the head of the new ministry.

"The principal objects of the King's speech were, to annul all the arrêtes of the Tiers Etat, from the time that they had constituted themselves into the National Assembly, and to establish the mode of voting *par ordre*, agreeably to the wishes of the majority of the noblesse and a great party among the clergy. It contains likewise a variety of most important concessions in favour of the people, which I have not time to mention to you at present, and concludes in the following words (for, unfortunately for His Majesty, the speech was immediately printed):—

"'Vous venez, Messieurs, d'entendre le resultat de mes dispositions et de mes vues : elles sont conformes au vif désir qui j'ai d'operer le bien public; et si par une fatalité loin de ma penseé, vous m'abandonniez dans une si belle entreprise seul je ferai le bien de mes peuples, seul je me considerai comme leur veritable representant; et connoissant vos cahiers, connoissant l'accord parfait qui existe entre le voeu le plus general de la nation et mes intentions bien faisantes j'aurai toute la confiance qui doit inspirer une si rare harmonie, et *je marcherai vers le but auquel je veux atteindre avec tout le courage et la fermeté qu'il doit m'inspirer!*

"'Réflechissez, Messieurs, qu'ancien de vos projets, aucune de vos dispositions, ne peut avoir force de loi sans mon approbation speciale. Ainsi je suis le garant naturel de nos droits respectifs, et tous les ordres de l'état peuvent se reposer sur mon equitable impartialité. Toute defiance de votre part seroit une grande injustice. C'est moi jusqú'à present qui fait tout pour le bonheur de mes peuples, et il est rare, peutêtre, que l'unique ambition d'un Souverain soit d'obtenir de ses sujets; qu'ils s'entendent enfin pour accepter ses bienfaits.

" 'Je vous ordonne, Messieurs, de vous separer tout de suite, et de vous rendre demain matin, chacun dans les Chambres apretées a votre ordre, pour y reprendre vos séances. J'ordonne en consequence au Grand Maitre des Ceremonies de faire preparer les salles.'

" After the King had retired, he sent two separate messages to the Deputies of the people to separate, and received for answer that nothing but force should determine them to leave the *Salle*, or to interrupt their deliberations. They then proceeded to pass a variety of resolutions, the chief of which were to persist in all their former arrêtes, and to continue their meetings as before, under the title of *Assemblée Nationale.*

" In the afternoon a great crowd assembled before the apartments of M. Necker (among whom were some hundreds of the Deputies of the people, and many people of very high rank), requesting him not to abandon them at so critical a juncture; and about five o'clock, *the very day that the above speech was pronounced*, he received a message from the King *and Queen*. He remained with them a considerable time, and at last came out and told the people he was to continue in office; for, in their anxiety to know the result, an immense crowd had forced their way into the apartments of the palace. He made an attempt to return to his own apartments by a private passage from the King's closet, but he was forced out of doors by the people, and, I am told, was carried home through a crowd who pressed forward to kiss his clothes. The same evening fireworks were thrown before his windows, and, it is said, continued the greater part of the night. I called accidentally next morning on a gentleman, who was in M. Necker's apartment during the whole of the business. It is supposed that this very extraordinary revolution, which took place so suddenly in the plans which the King seemed to have formed in the morning, proceeded from a discovery of the general dispositions of the army, and particularly of the Guards.

" Nothing of much consequence took place at Versailles the three following days. The Deputies of the people continued their *Séances*, under the title of *Assemblée Nationale*, and those of the other two orders in their own chambers. On the Wednesday (24th June), a considerable number of the noblesse, who had been restrained hitherto by the instructions they had received from their constituents, declared their resolution to unite themselves to the National Assembly, and the day following about fifty actually joined the Deputies of the Commons. Among these were the Duke of Orleans, the Comte de Montmorency, the Duc de la Rochefoucault, the Duc d'Aiguillon, the Comte de Clermot Tonerre, and a number besides of the very first names in France.

" *Despremenil*, and some others, particularly of the new noblesse, were still blustering in their own chamber. It is curious that *Despremenil*, who was a *roturier*, and who has not yet held his office the complete term of years necessary to confer *noblesse*, should have been on this occasion the great champion of his order, although he was frequently reminded of the history of his family by some of his brethren, in terms abundantly humiliating. It is said, in particular, that the Duc de Liancourt told him one day, in the Chamber of the Noblesse,—' Il faut avouer, Monsieur, que votre zele pour la noblesse soit bien desinterressé car il vous manque encore six mois d'etre Gentilhomme,' During this time the desertions of the clergy from their own Chamber to that of the Tiers Etat were continuing, insomuch that the President found one morning a bit of paper tied about the tongue of his bell (the Presidents of all the three orders have a small bell before them on the table, which they ring when they wish to establish order in their Assembly), with the following inscription,—*Vox clamans in deserto.*

" At last, on *Saturday*, the King sent a letter to the Chambers of the Clergy and the Noblesse, desiring them to unite themselves without delay to the National Assembly, which they accordingly did that very day.

" Since that time nothing of consequence has been done, as the Assembly has been wholly occupied *in verifying the powers* of the new members, and in some other preliminary business.

" I requested of you in a former letter, to take the trouble of mentioning the principal contents of my letters to D. Bannatyne and Mr Millar. Both of them will naturally expect that I should have written to them from time to time, and you will easily see the absolute impossibility of my doing so. I must therefore at present carry on my correspondence with them in this indirect manner, for which I beg you will make my apology. Remember me also to all my friends in Edinr.,—to whom I consider myself as writing when I address my letters to you. Best compts. to Mrs Alison, &c. &c.—I ever am, dear Archy, yours sincerely,

"D. S.

"*July* 2."

2. The Law of Aëriform Volumes extended to dense bodies. By J. G. Macvicar, A.M., D.D. Communicated by Professor Lyon Playfair.

In this communication the author proceeds to show that when dense bodies, whether liquids or solids, are regarded as consisting of certain molecules, the volumes of these molecules are either equals, halves, or doubles, &c., as in aëriforms.

This he proves by showing that the densities of bodies in general, as determined by the balance, are proportional to the numbers which express the weights of their molecules.

The author's theory of molecules is based on the following reasoning. By the general consent of men of science, nature is a dynamical system—a system of applied mathematics. The molecules of bodies are, when compared with the other properties of bodies, very stable clusters of atoms. In their structure, therefore, they may be expected to display in a high degree the geometrical and the dynamical, that is, the mechanical conditions of stability. These conditions in reference to individualised objects, each having but one centre, that is, in reference to such structures as the molecules of bodies, are most perfectly fulfilled, when the group of their parts or particles, regarded geometrically, is reducible to one or other of the regular polyhedrons. These polyhedrons are five in number; and of these, three (the tetrahedron, octohedron, and icosahedron) are of the same order, all having triangular faces; and all the three may be regarded as culminating in the last named and most perfect of the three, viz., the isocahedron. But the icosahedron, in its turn, is most intimately related to the dodecahedron, each under the application of the law of symmetry developing the other, each inscribing and circumscribing the other, giving both as alternate forms, as the radius lengthens or shortens. The remaining polyhedron, viz., the tube or hexahedron stands out from all the others, and is, in fact, singular, inasmuch as it has no proper poles, and seems unsuited for polarized action.

The author therefore fixes upon the dodecahedron and the icosahedron as the geometrical types of the molecules of bodies. He conceives that their molecules may possibly consist of 12 or of 20 chemical elements or units, either single units, binary units, ternary units, &c., as the case may be. And he proceeds to calculate solely from atomic weights, and without any reference whatever to atomic volumes, the densities of liquids and solids on this hypothesis.

But previously to entering upon details, he finds it necessary to allude to the law of molecular differentiation. Not that there is anything new in this law; for it is merely the further operation, when the stability of a molecule is threatened by heat or otherwise, of the grand principle of all chemical synthesis, viz., the concurrence and apposition of dissimilar particles; but it modifies the number of elements in a molecule according to the risks which that molecule has run; and consequently it modifies also the specific gravity of the mass. Thus taking X to represent any chemical element; instead of simple dodecatoms X_{12} or icosatoms X_{20}, both of which are isometrical as well as homogeneous, we may have a composite molecule, $X_{12}X_{20}=X_{32}$, consisting of both, the one differentiating the other, and thus securing a greater stability for both. Similarly single elements of X may differentiate both X_{12} and X_{20}, giving $X\ X_{12}X=X_{14}$ and $X\ X_{20}X=X_{22}$. And these two, in their turn, may differentiate each other, giving $X\ X_{12}X,\ X\ X_{20}X = 36X$. And here one of the limits of the theory as a method of reaching the construction of molecules presents itself. Thus the compound undifferentiated dodecatom $(X_{12})_{12}$, occupying four volumes, must give the same specific gravity as $X\ X_{12}XX\ X_{20}X = 36X$, occupying one volume; for $\frac{12 \times 12\ X}{4} = 36X$.

To obtain densities in the familiar form of specific gravities the molecule or unit volume of water requires to be determined. And this the author finds to be a dodecatom (or rather an isobaric molecule), each constituent element of which is 3HO

$$=36\ \mathrm{aq}=\mathrm{AQ}=36\times 9=324 \text{ when } \mathrm{H}=1.$$

He then proceeds to deduce theoretically the specific gravities of between seventy and eighty of the most interesting and abundant of natural and chemical substances, somewhat in the order in which they are treated in chemical works.

A composite molecule, consisting of the two isometrical molecules X_{12} and X_{20}, differentiating each other, and giving $X_{12}X_{20}=X_{32}$, gives the idea of great molecular perfection and stability—as, for instance, repose in the presence of oxygen, &c. He finds, accordingly, that such is the molecule of the precious metals, or metals that remain pure, and the diamond; gold and silver having normal or aqueous volumes, platinum and aluminium half, and bismuth and antimony double volumes; the platinum and antimony molecules being also differentiated, and therefore their molecular structures equivocal. Thus—

$$\text{Gold } G=\frac{Au_{12}Au_{20}}{AQ}=\frac{197\times\overline{12+20}}{324}=19{\cdot}1. \quad \text{Expt. } 19{\cdot}2.$$

$$\text{Aluminium } G=\frac{2(Al_{12}Al_{20})}{AQ}=\frac{2(13{\cdot}75\times\overline{12+20}}{324}=2{\cdot}7. \quad \text{Expt. } 2{\cdot}6\,.\,2{\cdot}7.$$

The diamond, by the greater openness of X_{20} than X_{12} (when both are formed of pentagonal elements), and its more ready combustibility, almost enables us to demonstrate analytically this composite molecule of $X_{12}X_{20}$. In so light an element as carbon, however, when constructed by nature, it is not the single element $C_{12}C_{20}$, but the dodecatom $(C_{12}C_{20})_{12}$ that is in relation with the unit volume of water. Thus—

$$\text{Diamond } G=\frac{(C_{12}C_{20})_{12}}{2AQ}=\frac{12\times 6\times\overline{12+20}}{2\times 324}=3{\cdot}55. \quad \text{Expt. } 3{\cdot}55.$$

$$\left.\begin{array}{l}\text{Unburnt}\\ \text{residuum,}\end{array}\right\} G=\frac{(C_{12})_{12}}{AQ}=\frac{12\times 6\times 12}{324}=2{\cdot}67. \quad \text{Expt. } 2{\cdot}67 \text{ (Jaquealine).}$$

$$\text{Coke and Graphite? } G=\frac{(C_{20})_{12}}{2AQ}=\frac{12\times 6\times 20}{2\times 324}=2{\cdot}22. \quad \text{Expt. } 1{\cdot}8\,..\,2{\cdot}3.$$

Similarly with phosphorus. White or old $\frac{P_{12}P_{20}}{2AQ}=1{\cdot}53$. Least combustible $\frac{2P_{12}}{AQ}=2{\cdot}29$. Most combustible $\frac{P_{20}}{AQ}=1{\cdot}913$.

The dodecatom generally has half the volume of the icosatom of the same substance. When mercury solidifies, it appears to change from the more open to the more compact. Thus

$$\text{Mercury } G = \begin{cases} \dfrac{2(Hg)}{AQ}=\dfrac{2(100\times 22)}{324}=13{\cdot}58. & \text{Expt. } 13{\cdot}59 \text{ (at Zero.)} \\[2ex] \dfrac{4(Hg)_{12}}{AQ}=\dfrac{4(100\times 12)}{324}=14{\cdot}8. & \text{Expt. } 14{\cdot}4. \text{ (Solid.)} \end{cases}$$

Lithium, sodium, potassium, are $X\ X_{12}X$; sodium occupying a normal volume; lithium, calcium, magnesium, a half; and potassium double volume.

$$\text{Lithium } \frac{2Li_{14}}{AQ}=\frac{2(7\times 14)}{324}=\cdot 60. \quad \text{Expt. } \cdot 59.$$

$$\text{Sodium } \frac{Na_{14}}{AQ}=\frac{23\times 14}{324}=\cdot 99. \quad \text{Expt. } \cdot 97.$$

$$\text{Potassium } \frac{K_{14}}{2AQ}=\frac{39\times 14}{2\times 324}=\cdot 84. \quad \text{Expt. } \cdot 86.$$

And so on.

$$\text{Bromine } \frac{Br_{12}}{AQ}=\frac{80\times 12}{324}=2{\cdot}99. \quad \text{Expt. } 2{\cdot}99.$$

$$\text{Table Salt } \frac{(NaCl)_{12}}{AQ}=\frac{12\times\overline{23+35{\cdot}5}}{324}=2{\cdot}16. \quad \text{Expt. } 2{\cdot}15.$$

$$\text{Quartz } \frac{2(SiO_2)_{14}}{AQ}=\frac{2(30\times 14)}{324}=2{\cdot}59. \quad \text{Expt. } 2{\cdot}5\ \ldots\ 2{\cdot}8.$$

$$\text{Sulphur } \frac{S_{20}}{AQ}=\frac{32\times 20}{324}=1{\cdot}975. \quad \text{Expt. } 1{\cdot}98.$$

$$\left.\begin{array}{c}\text{Glacial}\\ \text{sulphuric}\\ \text{acid}\end{array}\right\} \frac{(SO_3+2HO)_{20}}{2AQ}=\frac{20(40+18)}{2\times 324}=1{\cdot}79. \quad \text{Expt. } 1{\cdot}785 \text{ (Playf.)}$$

$$\left.\begin{array}{l}\text{Most stable hydrate}\\ \text{of chlorhydric acid,}\end{array}\right\}G=\frac{2(HCl+16aq)}{324}=1{\cdot}111. \quad \text{Expt. } 1{\cdot}111.$$

All molecules determined by hydrogen and sulphur are icosatoms, others very generally are dodecatoms.

And so on, with between 70 and 80 familiar substances.

Potass-soda felspar is a dodecatom of Potass felspar, viz.

$$(KOAl_2O_3(SiO_2)_{12}\,O_3Al_2OK)_{12}$$

in which in the two polar elements, KO is substituted by NaO; occupying eight normal volumes as such highly composite molecules usually do.

			Theory.	Abich.	
Felspar	144 SiO_2	4300	65·14	65·72	From Baveno.
	24 Al_2O_3	1248	18·82	18·57	
	20 KO	940	14·17	14·02	
	4 NiO	124	1·25	1·25	

$G=8\times324)\overline{6632}(2{\cdot}55.$ The balance gives 2·5552.
See *Millar's Miner.*, p. 367.

3. On the Freezing of the Egg of the Common Fowl. By John Davy, M.D., F.R.S. Lond. and Edin.

The author, preparatory to stating the experiments which he has made on the freezing of the egg of the common fowl, referred to the early ones of Mr Hunter on the same subject, and to the later ones of Mr Paget, both these inquirers agreeing as to the results, but differing in their conclusions, Hunter attributing the resistance which the egg offers to freezing to a supposed vital principle, Mr Paget attributing it to the peculiar viscid state of the albumen.

The author, from his results, infers that two causes are mainly operative in protecting the egg; one, the cellular filamentous structure in which the albumen is contained; the other, the peculiar composition of the contents of the egg, and especially their saline elements. He concludes by proposing some queries respecting life, whether it can exist without action, and whether, provided the organic structure is not injured, congelation can take place without the death of the part frozen.

4. On the Variations of the Fertility and Fecundity of Women according to Age. By Dr J. Matthews Duncan.

This paper was divided into four parts. In the first part it was shown :—

1. That the actual, not the relative, fertility of our female population, as a whole, at different ages, increases from the commencement of the child-bearing period of life until the age of 30 is reached, and then declines to its extinction with the child-bearing faculty.

2. That the actual fertility is much greater before the climax, 30 years, is reached, than after it is passed.

3. That at least three-fifths of the population are recruited from women not exceeding 30 years of age.

In the second part it was shown :—

1. That comparative fertility increases gradually from the commencement of the child-bearing period of life until about the age of 30 years is reached, and that then it still more gradually declines.

2. That it is greater in the decade of years following the climax of about 30 years of age than in the decade of years preceding the climax.

In the third part it was shown :—

1. That the fecundity of the mass of wives in our population is greatest at the commencement of the child-bearing period of life, and after that epoch gradually diminishes.

2. That the fecundity of the whole wives in our population included within the child-bearing period of life is, before 30 years of age is reached, more than twice as great as it is after that period.

3. That the fecundity of the wives in our population declines with great rapidity after the age of 40 is reached.

In the fourth part it was shown :—

1. That the initial fecundity of women gradually waxes to a climax and then gradually wanes.

2. That initial fecundity is very high from 20 to 34 years of age.

3. That the climax of initial fecundity is probably about the age of 25.

5. On the Chemical and Physical Principles in Connection with the Specific Gravity of Liquid and Solid Substances. By Otto Richter, Ph. D. Communicated by Professor Maclagan.

The subject of this paper, is of a nature to find favour with all those chemists who are disposed to recognise in the fast accumulating evidence of a new and remarkable order of chemical phenomena rich and valuable materials for the construction of a sounder and more comprehensive chemical theory. This new order of phenomena appears to me, nevertheless, but imperfectly appreciated and understood, even on the part of our most distinguished experimentalists. Thus, for instance, Monsieur Pasteur, the illustrious French philosopher, and with him a host of other eminent thinkers, is of opinion, that matter is indebted for its chemical and physical properties mainly and exclusively to the peculiar manner in which the atoms are grouped together. In alluding to the probable cause of circular polarisation, Monsieur Pasteur considers, that this singular property ought to be referred to the unsymmetrical disposition of the constituents, and that the restoration of the chemical symmetry must at the same time obliterate every trace of circularly polarising power. Again, in order to account for the fact, that certain salts, for instance the chlorate of soda, cease to exhibit circular polarisation so soon as they are made to pass from the solid into the liquid state, he ventures even a step farther. According to him these optical phenomena are really due to two distinct causes, either to the dissymmetrical distribution of the atoms within the limits of each individual molecule, or to the spiral form of aggregation impressed upon entire masses of molecules, although these latter may, as individuals, exist in a state of chemical symmetry and optical neutrality. Monsieur Pasteur adopts the following mode of reasoning:—The chlorate of soda owes its optical activity mainly and exclusively to the spiral form of aggregation; and being destitute of that property, so far as its individual molecules are concerned, it follows, that the destruction of the spiral arrangement during the process of solution, must leave the chlorate of soda optically, because constitutionally, passive.

This argument, which *prima facie* seems so ingenious and plausible, is, nevertheless, in my opinion, void of foundation, because it rests on premises which are *empirically* and *philosophically* inadmissible. In the first place, *empirically*, because the hypothesis postulates, that a ray of light is propagated, both within and beyond the polarising medium, in the form of a spiral; whereas Monsieur Fresnel, another distinguished French philosopher, has clearly proved, both experimentally and mathematically, that the plane of polarisation assumes this twisted form in virtue of the mutual interference of two pencils of light, into which the original beam has resolved itself, and the particles of which, instead of vibrating as formerly in straight lines, are henceforth made to vibrate in the direction of a curve, and across two opposite regions of the influencing molecule. In the second place, *philosophically*, because the hypothesis places us between the horns of the following dilemma: either to make the optical change dependant upon a purely *mechanical* cause, namely, upon the assumption of a pre-existing spiral form of arrangement, in consequence of which a beam of light is *nolens volens* compelled to travel along the solid walls of a winding tunnel, or to make the optical change dependant upon a purely *dynamical* cause, namely, upon the assumption of a pre-existing unsymmetrical disposition of the atoms within the limits of each individual molecule; in consequence of which the molecule is supposed all at once to acquire the power of turning the luminous particles from their rectilinear course.

Now, it is a well known axiom of cosmical economy, that, in trying to interpret facts, we ought not to multiply causes unnecessarily, and that one and the same class of phenomena ought, if possible, to be referred to one cause only; but when, as in the case before us, these causes are so essentially distinct, both in their form and in their mode of action, the arguments employed by Monsieur Pasteur, unsupported as they are by either experiments or calculations, appear to me no longer tenable, and it behoves us, therefore, to search for a more consistent and comprehensive explanation. In reviewing the new order of phenomena above alluded to, and which embraces the following principal physical properties of matter,—viz. specific gravity, specific volume, allotropism and polymorphism, fusing and boiling points, crystalline form, optical, electrical, and magnetic polarity, &c., the general aspect of this order of facts seems to me so marked and peculiar, and at the same time so unchemical, that I consider myself justified in vindicating, for their scientific conception and interpretation, the existence of a principle *sui generis*,—a principle which, in virtue of its characteristic mode of action, I hold to be capable of modifying the chemical and physical energies of every part of a given atomic system, and all this, be it well observed, with the entire preservation of the original arrangement. Let me, in illustration of my meaning, assume, in the first place, that the so-called atoms,—a definite number of which I conceive to be always and inseparably united according to some fixed principle of grouping, and thus to constitute an elementary atomic system or molecule,—are endowed with repulsive energies directly proportional to the range and intensity of their own vibratory movements, which latter may be supposed to consist in a series of periodical expansions and contractions of each spheroidal atom; let me assume, in the second place, that under the influence of a certain physical agent the whole, or a given portion, of the atoms composing a molecule may experience a suspension or restoration of their vibratory movements; and let me assume, in the last place, that the specific volumes of substances in the liquid and solid state depend upon and correspond to the degree of repulsive energy thus developed in contiguous conflicting molecules; then, by means of these three assumptions, we have established a *nexus causalis* between the variations in the specific volume of dense molecules and between the intensity of their repulsive energies. Now it seems to me very clear that, so long as the peculiar force, which causes these variations, and which I will provisionally distinguish by the term *paralytic force*, happens to induce precisely the same changes upon any two opposite sides of a given atomic system, the physical symmetry of that system will not be disturbed, but, on the contrary, that, when these changes differ, physical symmetry can no longer exist, and that the natural result will be the formation of an ethereal current from the stronger towards the weaker side during the act of atomic expansion, and of another current in the contrary direction during the act of atomic contraction. Applying this view to the phenomena of circular polarisation, I have no hesitation in maintaining, that they have their origin in physical dissymmetry only, and that the existence of the vaunted chemical dissymmetry is at least very problematical. The reason, therefore, why chlorate of soda and its congeners lose the property of circular polarisation during the act of solution must be sought in differences of temperature and the altered state of aggregation, both of which conspire in restoring the physical symmetry, while the process of crystallisation tends to produce the opposite effect. I may add, moreover, that the discovery of a substance exhibiting the same optical deportment as chlorate of soda, while these physical processes are reversed, would prove a powerful argument in support of my view, but utterly subversive of Monsieur Pasteur's mode of reasoning.

The theory here advocated naturally leads us to suspect the existence of another set of equivalents, namely, the volume equivalents; their numerical expression would correspond to the highest degree of repulsive energy, and to the greatest amount of vibratory movements, of which the various species of elementary molecules are susceptible. They would, however, differ materially from the weight equivalents, which are constant, whereas they would, on the contrary, be subject to a peculiar law of variations, which enables them to pass through a series of values from a given maximum down to zero. I am not at present prepared to enter into details, but a careful comparison of facts has convinced me that the variations in the specific volumes of the molecules, or, what comes to the same thing, the variations in the volume-equivalents of their component elements, are genetically connected, and run parallel with the modifications of their various physical properties. If, then, all the chemical and physical properties of matter are due to the simultaneous agency of two essentially distinct principles, it follows that chemistry, both as an art and as a science, rests not on a simple but on a complex foundation. On the one hand, it is the purely chemical principle which determines the various forms of molecular arrangement, and it is the ponderable portion alone of the various kinds of atoms, which, in their constant weight-equivalents, furnishes the basis of calculation. On the other hand, it is the purely physical principle which determines the specific volume of the molecules, and it is the imponderable portion alone of the various kinds of atoms, which, in their variable volume-equivalents, furnishes the basis of calculation. Accordingly, our tree of chemical knowledge splits up into two main branches, to which we might apply the terms Pondo-chemistry and Impondo-chemistry. I beg to remark, in conclusion, that having resolved upon a minute and thorough investigation of this, no doubt very intricate, subject, I was surprised to find, after examining all the principal sources of information, that a complete and trust-worthy catalogue of the leading physical properties of matter was still a desideratum, but more particularly that the determination of the specific gravities of substances in the liquid and solid state had been much neglected. With the most ample resources at my disposal, I have at length succeeded in composing a catalogue of these latter. The number of *bona fide* cases entered therein amounts to about 1800; but this number is after all a mere fraction, in comparison with the hundreds of thousands of non-determined bodies. I may also state, that should a systematic revision of this most needful portion of our science be resolved upon, I shall feel great pleasure in presenting my catalogue, such as it is, to the parties entrusted with its execution.

6. On the Theory of Isomeric Compounds. By Dr A. Crum Brown.

In this paper only those bodies are considered which are "absolutely isomeric," that is, which have not only the same composition per cent. and the same molecular weight, but also the same constitutional formula. As the constitutional formula of comparatively few substances is known, this class is as yet a small one.

The following pairs of substances are probably absolutely isomeric.

1. The alcohol radicals and the hydrides, as methyl gas and the hydride of ethyl.
2. Chloride of ethyl and the product of the action of chlorine on hydride of ethyl.
3. Chloride of vinyl and chloracetene.
4. Fumaric and maleic acids.

5. Bromomaleic and isobromomaleic acids.
6. The two bibromosuccinic acids.
7. The two varieties of malic acid.
8. The two varieties of aspartic acid.
9. Two of the varieties of tartaric acid.
10. Two of the dehydrogenates of pyrotartaric acid.
11. Two of the three bibromopyrotartaric acids.

The following are shown to be probably *metameric*,—

1. The compounds of ethylene and of ethylidene.
2. The alcohols proper and the hydrates of the olefines.
3. Lactic and paralactic acids.

The next point considered is the influence which the existence of such pairs of substances has on the theory of atomicity, and particularly on the question whether all the equivalents of a multequivalent atom are of the same chemical nature.

The consideration of absolutely isomeric bodies leads to the conclusion that there is a difference among the equivalents of the same atom, and that this difference is not entirely due to the structure of the molecule of which the atom forms a part. It is not, however, possible as yet to apply this principle to the explanation of particular cases.

The remarks of Professor Kekulé on the isomerism of the dehydrogenates of succinic and of pyrotartaric acids, and those of Professor Butlerow on the isomerism of methyl gas and hydride of ethyl, are then considered in detail, and Butlerow's hypothesis compared with the views of Kolbe as to the chemical nature of carbon.

7. On the Rhombohedral System in Crystallography. By Alfred R. Catton, B.A., Scholar of St John's College, Cambridge, and Fellow of the Cambridge Philosophical Society. Communicated by Professor Tait.

1. The science of Crystallography has for its object the classification of crystalline forms.

The principles which must guide us in attempting to arrive at a natural classification of crystals are the same as those which have been so successfully applied to the formation of a natural classification of plants by Ray and Jussieu, and of animals by Cuvier.

By the application of these principles it has been found that crystals are formed on six different types or plans of structure, and in consequence crystals have been arranged in six primary classes, termed crystallographic systems. They are named the cubic, pyramidal, rhombohedral, prismatic, oblique, and anorthic systems.

It is the object of this paper to show that the type on which crystals of the rhombohedral system are constructed is the same as that of the prismatic system.

The establishment of this proposition necessitates the abolition of the rhombohedral system, and all crystals hitherto included in it must in future be referred to the prismatic system.

2. In order to establish this proposition it is necessary to show,

(1.) That three axes of symmetry can be found at right angles to each other, such that for them the crystallographic parameters become unequal.

(2.) That the laws of symmetry of crystals of the rhombohedral system are the same as those of the prismatic system.

Now it is easily seen that the three straight lines perpendicular to the planes $10\bar{1}$, $\bar{1}2\bar{1}$, and 111 are at right angles to each other.

For the forms $10\bar{1}$ and $2\bar{1}\bar{1}$ differ only in position, the one being the same as the other turned through an angle of 30^0 about the optic axis.

Also the faces of the form 111 are perpendicular to the faces of each of the above forms. The three straight lines perpendicular respectively to the faces $10\bar{1}$, $\bar{1}2\bar{1}$, and 111, are taken as the new axes of symmetry; the parameters corresponding to them being a, b, c, respectively, it is shown in a later part of the paper that a, b, c, are unequal.

3. The paper then proceeds to show that the laws of symmetry of crystals of the rhombohedral system are the same as those of the prismatic system.

This is done by taking each simple form separately and finding what the indices of its faces become when referred to the new axes; it is then found that the new indices for all the simple forms follow the laws of symmetry of crystals of the prismatic system.

The same process is followed with the hemihedral forms.

4. In order to find the indices of a given face referred to the new axes, it is necessary to solve the following problem :—

"Given the indices and parameters of any face of a crystal when referred to given axes, to find its indices and also the new parameters when referred to any other axes originating in the same point."

This problem is solved in the paper, but the solution is too long to be here given.

For the purposes of this paper only a particular case of the general problem is required.

For in the rhombohedral system the axes make equal angles with each other, and the parameters are equal. Let ω be the angle between the rhombohedral axes, and a the magnitude of each of the equal parameters. Let hkl be the symbol of a face of a crystal belonging to the rhombohedral system, and $h'k'l'$ the indices of this face when referred to axes perpendicular respectively to the faces $10\bar{1}$, $\bar{1}2\bar{1}$, 111, a', b', c', the new parameters.

Then the formulæ obtained in the general case become, in this instance,—

$$\left.\begin{aligned} h' &= \frac{h-l}{2} \\ k' &= \frac{2k-h-l}{6} \\ l' &= \frac{h+k+l}{3} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1.)$$

These expressions give the new indices.

The new parameters are given by the following formulæ.

$$a'x = \sin \tfrac{1}{2}\,\omega$$

$$b'x = \frac{1}{\sqrt{3}} \sin \tfrac{1}{2}\,\omega$$

$$c'x = \left\{ \frac{1 + 2\cos \omega}{3} \right\}^{\frac{1}{2}}$$

Where x is a constant.

5. Let α be the "angular element" of a crystal of the rhombohedral system, *i.e.* the angle between normals from the origin to the faces 100 and 111.

Then it is shown in the paper, that the expressions for the new parameters become,—

$$a'x = \left\{ \frac{3}{4 + \tan^2 \alpha} \right\}^{\frac{1}{2}}$$

$$b'x = \frac{1}{\{4 + \tan^2 \alpha\}}$$

$$c'x = \frac{\tan \alpha}{\{4 + \tan^2 \alpha\}^{\frac{1}{2}}}$$

Hence, a', b', c', are respectively proportional to $\sqrt{3}$, 1, $\tan \alpha$;

$$\therefore \quad \left. \frac{a'}{\sqrt{3}} = \frac{b'}{1} = \frac{c'}{\tan \alpha} \right\} \quad . \quad . \quad . \quad . \qquad (2.)$$

Hence, given the angular element and the indices of a face, the ratios of the new parameters are determined by the formulæ (2.), and the new indices by the formulæ (1.)

Ex.—Find the new indices of the faces $01\bar{1}$, $13\bar{1}$.

α. The face $01\bar{1}$.

$$h' = \frac{0+1}{2}, \quad k' = \frac{2-0+1}{6}, \quad l' = \frac{0+1-1}{3}$$

$$= \tfrac{1}{2} \qquad\quad = \tfrac{1}{2} \qquad\quad = 0$$

$\therefore$ the new symbol is 110.

β. The face $13\bar{1}$.

$$h' = \frac{2}{2}, \quad k' = \frac{6-1+1}{6}, \quad l' = \frac{1+3-1}{3}$$

$$= 1 \qquad\quad = 1 \qquad\quad = 1.$$

$\therefore$ the new symbol is 111.

6. Also, given the symbol of a simple form belonging to the rhombohedral system, and also the angular element, the indices

of its several faces when referred to the new axes can be determined, and also the ratios of the new parameters.

The simple forms of the rhombohedral system are then taken in succession, and the indices of their faces when referred to the new axes are found. It is then seen that the new indices follow in all cases the laws of symmetry of the prismatic system.

This is done at length in the paper. The general process is exemplified here by its application to one or two simple forms only.

7. The simple form $10\bar{1}$, consists of the following faces.

$$\begin{array}{ccc} 10\bar{1} & 0\bar{1}\bar{1} & \bar{1}10 \\ \bar{1}01 & 0\bar{1}1 & 1\bar{1}0 \end{array}$$

By the formulæ (1.), it is found that the new indices are, respectively,

$$\begin{array}{ccc} 100 & 110 & \bar{1}10 \\ \bar{1}00 & \bar{1}\bar{1}0 & 1\bar{1}0 \end{array}$$

These indices are divisible into two groups,

$$\left\{\begin{array}{cc} 110 & \bar{1}10 \\ \bar{1}\bar{1}0 & \bar{1}10 \end{array}\right.$$

$$\text{And} \left\{\begin{array}{c} 100 \\ \bar{1}00 \end{array}\right.$$

Now, it will be observed that in the first group, the indices always occur in the order 1, 1, 0; and that the symbols of the faces consist of every possible arrangement of the symbols $\pm 1, \pm 1, 0$, in which unity occupies the first and second places, and zero the last.

The faces of the first group, therefore, follow the law of symmetry of the prismatic system, and belong to the form 110.

Similarly, the faces

$$\begin{array}{c} 100 \\ \bar{1}00 \end{array}$$

belong to the form 100.

Hence, the form $10\bar{1}$ of the rhombohedral system, is a combination of the forms 110 and 100 of the prismatic system.

In a similar manner it can be shown, that the form 211, is a combination of the forms 310 and 010.

8. The form hkl, where one of the indices is an arithmetical mean between the other two, is a double six-faced pyramid, whose base is perpendicular to the optic axis.

First, let $2h=k+l$, or $h-k=l-h$.

The faces may be arranged in the following two groups:—

$$\left.\begin{matrix} hlk & lkh & klh & hkl \\ \bar{h}\bar{l}\bar{k} & \bar{l}\bar{k}\bar{h} & \bar{k}\bar{l}\bar{h} & \bar{h}\bar{k}\bar{l} \end{matrix}\right\} \quad . \quad . \quad . \quad \text{(A.)}$$

$$\left.\begin{matrix} lhk & khl \\ \bar{l}\bar{h}\bar{k} & \bar{k}\bar{h}\bar{l} \end{matrix}\right\} \quad . \quad . \quad . \quad . \quad \text{(B.)}$$

Let the new indices of the face hlk be $\alpha\beta\gamma$.

Then $$\alpha=\frac{h-k}{2}, \quad \beta=\frac{2l-h-k}{6}, \quad \gamma=\frac{h+l+k}{3}.$$

Now, $$\frac{2l-h-k}{6}=\frac{4h-2k-h-k}{6}=\frac{h-k}{2}$$

$$\therefore \beta=\alpha.$$

And $$\frac{h+k+l}{3}=\frac{h+2h}{3}=h.$$

Hence the face hlk becomes $\alpha\alpha h$, where $\alpha=\dfrac{h-k}{2}$.

Again, let the new indices of the face lkh be $\alpha'\beta'\gamma'$.

Then $$\alpha'=\frac{l-h}{2}, \quad \beta'=\frac{2k-l-h}{6}, \quad \gamma'=\frac{l+k+h}{3}$$
$$=\frac{h-k}{2} \qquad =\frac{k-h}{2} \qquad =h.$$

Hence the face lkh becomes $\alpha\bar{\alpha}h$.

Similarly, the new indices of the faces klh, hkl are $\bar{\alpha}\alpha h$ and $\bar{\alpha}\bar{\alpha}h$ respectively.

Hence the group of faces, (A), becomes

$$\left.\begin{matrix} \alpha\alpha h, & \alpha\bar{\alpha}h, & \bar{\alpha}\alpha h, & \bar{\alpha}\bar{\alpha}h \\ \bar{\alpha}\bar{\alpha}\bar{h}, & \bar{\alpha}\alpha\bar{h}, & \alpha\bar{\alpha}\bar{h}, & \alpha\alpha\bar{h} \end{matrix}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A$'$.)}$$

where $\alpha=\dfrac{h-k}{2}$; if $h-k$ be odd, each of the indices in (A′) must be multiplied by two.

Again, let the new indices of the face lhk be $\alpha'\beta'\gamma'$,

$$\text{Then } \alpha'=\frac{l-k}{2}, \quad \beta'=\frac{2h-l-k}{6}, \quad \gamma'=\frac{l+h+k}{3}$$
$$=h-k \qquad =o \qquad =h.$$

Hence the face lhk becomes $2\alpha\ o\ h$.

Similarly, the face khl becomes $\overline{2\alpha}\ o\ h$.

Hence the group of faces (B) becomes,

$$\left.\begin{array}{l} 2\alpha\ o\ h,\ \bar{2}\alpha\ o\ h \\ \bar{2}\alpha\ o\ \bar{h},\ 2\alpha\ o\ \bar{h} \end{array}\right\} \quad . \ . \ . \ . \ . \ . \ . \quad (\mathrm{B}').$$

Now it will be observed that, in the group (A′), the indices always occur in the order $\alpha\alpha h$; and that the symbols of the faces consist of every possible arrangement of the indices $\pm\alpha$, $\pm\alpha$, $\pm h$, in which α occupies the first and second places and h the last. The faces of the group (A′) are therefore subject to the law of symmetry of the prismatic system, and belong to the form

$$\alpha\alpha h.$$

Similarly the faces of the group (B′) belong to the form

$$2\alpha\ o\ h.$$

Hence the form hkl of the rhombohedral system, where $2h = k + l$ is a combination of the forms $\alpha\alpha h$ and $2\alpha\ o\ h$ of the prismatic system, where $\alpha = \frac{h-k}{2}$.

If $2k = l + h$, it can be shown in a similar manner that hkl is a combination of the forms $\beta\beta k$ and $2\beta\ o\ k$, where $\beta = \frac{k-l}{2}$.

And if $2l = h + k$, a combination of the forms $\gamma\gamma l$ and $2\gamma\ o\ l$, where $\gamma = \frac{l-h}{2}$.

Ex.—Transfer the forms $13\bar{1}$ and $17\bar{5}$ to the prismatic system.

α. The form $13\bar{1}$.

$$\text{Here } h = 1,\ k = -1,\ \therefore \alpha = \frac{2}{2} = 1,$$

$\therefore$ $13\bar{1}$, is a combination of 111 and 201.

β. The form $17\bar{5}$.

$$\text{Here } h = 1,\ k = -5,\ \therefore \alpha = \frac{1+5}{2} = 3,$$

$\therefore$ $17\bar{5}$ is a combination of 331 and 601.

9. The results of the examination of the other simple forms of the rhombohedral system will be given without demonstration.

The form hkl, where $h+k+l=o$ is a twelve-sided prism, whose axis is parallel to the optic axis.

It is a combination of the following forms of the prismatic system :—

$$\alpha_1\beta_1 o,\ \ \alpha_2\beta_2 o,\ \ \alpha_3\beta_3 o,$$

$$\text{where } \alpha_1=\frac{k-l}{2},\ \beta_1=\frac{2h-k-l}{6},$$

$$\alpha_2=\frac{l-h}{2},\ \beta_2=\frac{2k-l-h}{6},$$

$$\alpha_3=\frac{h-k}{2},\ \beta_3=\frac{2l-h-k}{6}.$$

Ex.—The form $3\bar{1}\bar{2}$ is a combination of 210, 130, 510.

10. The form hkk is a rhombohedron.

It can be shown that the rhombohedra

$$hkk,$$

$$\text{and } 4k-h,\ \ 2h+k,\ \ 2h+k,$$

are inverse with respect to each other.

A combination of these rhombohedra is a combination of the following forms of the prismatic system—

$$3\alpha\ \alpha\ 2\beta;\ o\ \alpha\ \beta,$$

$$\text{where } \alpha=h-k\quad \beta=2k+h.$$

Ex.—A combination of the rhombohedra 100 and $\bar{1}22$ (which are inverse with respect to each other), is a combination of the forms 312 and 011 of the prismatic system.

The rhombohedron hkk itself is a combination of two hemihedral forms with parallel faces, viz.,—

$$\pi\ 3\alpha\ \alpha\ 2\beta \text{ and } \pi\ o\alpha\beta,$$

where α and β have the same values as above.

11. The form hkl, where h,k,l are connected by none of the preceding relations, is sometimes termed a scalenohedron.

It can be shown that the following scalenohedra are inverse with respect to each other, viz.,—

$$hkl$$

$$\text{and } 2k+2l-h,\ \ 2l+2h-k,\ \ 2h+2k-l.$$

A combination of these scalenohedra is a combination of the following forms of the prismatic system,—

$$\alpha_1\beta_1\gamma, \ \alpha_2\beta_2\gamma, \ \alpha_3\beta_3\gamma,$$

$$\text{where } \alpha_1 = \frac{k-l}{2}, \ \beta_1 = \frac{2h-k-l}{6},$$

$$\alpha_2 = \frac{l-h}{2}, \ \beta_2 = \frac{2k-l-h}{6},$$

$$\alpha_3 = \frac{h-k}{2}, \ \beta_3 = \frac{2l-h-k}{6},$$

$$\gamma = \frac{h+k+l}{3}.$$

Ex.—The scalenohedra $16\bar{1}$ and $35\bar{2}$, which occur in apatite, are inverse with respect to each other.

A combination of these scalenohedra is a combination of the forms 122, 534, 714 of the prismatic system.

The scalenohedron *hkl* itself is a combination of three hemihedral forms with parallel faces, viz.,—

$$\pi\alpha_1\beta_1\gamma, \ \pi\alpha_2\beta_2\gamma, \ \pi\alpha_3\beta_3\gamma,$$

where $\alpha_1\beta_1$, $\alpha_2\beta_2$, $\alpha_3\beta_3$, and γ have the same values as above.

12. The simple forms of the rhombohedral system, therefore, with the exception of the rhombohedron *hkk*, and the scalenohedron *hkl*, follow the law of symmetry of the holohedral forms of the prismatic system. Hence it follows that the hemihedral forms also, derived from these simple forms, are subject to the laws of symmetry of the hemihedral forms of the prismatic system.

Since the rhombohedron and scalenohedron are themselves combinations of hemihedral forms with parallel faces of the prismatic system, it follows that the hemihedral forms derived from the rhombohedron and scalenohedron are combinations of *tetartohedral* forms, with inclined and parallel faces, derived from the hemihedral forms with parallel faces of the prismatic system.

These tetartohedral forms with inclined and parallel faces, derived from the hemihedral forms with parallel faces, may be denoted respectively by the symbols $\kappa\pi hkl$ and $\pi\pi hkl$.

The tetartohedral form $\pi\kappa hkl$ cannot obviously exist; and it is easily seen that the form $\kappa\kappa hkl$ is included in $\kappa\pi hkl$. Hence the only tetartohedral forms are $\pi\pi hkl$ and $\kappa\pi hkl$,—*i.e.*, they are all derived from the hemihedral forms with parallel faces.

Also $\pi\pi hkl$ can only exist when all the indices h, k, l are finite, and $\kappa\pi hkl$ can only exist when all the indices are finite or one zero.

When hkl are all finite, $\pi\pi hkl$ consists of any pair of parallel faces of the form hkl and $\kappa\pi hkl$ of any pair of inclined faces of the form hkl. When one of the indices is zero, $\kappa\pi hkl$ consists of any single face of the form hkl.

13. Hemihedral forms with inclined faces, derived from the rhombohedron or scalenohedron, have only been observed in the following minerals,—

Cronstedtite.
Phenakite.
Pyrargyrite.
Quartz.
Tourmaline.

In these minerals alone, therefore, do the tetartohedral forms $\kappa\pi hkl$ occur.

Hemihedral forms with parallel faces, derived from the scalenohedron, have only been observed in the following minerals,—

Dolomite.
Dioptase.
Phenakite.

The tetartohedral forms $\pi\pi hkl$ occur, therefore, only in these minerals.

Hemihedral forms with parallel faces, derived from scalenohedra, occur also in apatite. But in this mineral, two scalenohedra, inverse with respect to each other, always occur together on the same specimen, and when one scalenohedron is hemihedral the other is so also. Hence the hemihedral forms in this mineral correspond to hemihedral and not to tetartohedral forms in the prismatic system.

14. Let α_x, α_y, α_z be the new *angular elements* of crystals of the rhombohedral system when transferred to the prismatic system.

$$\text{Let } \alpha_x = 100, 110; \quad \alpha_y = 010, 011; \quad \alpha_z = 001, 101.$$

$$\text{Then } \tan \alpha_x = \frac{a'}{b'}, \quad \tan \alpha_y = \frac{b'}{c'}, \quad \tan \alpha_z = \frac{c'}{a'};$$

And by the formulæ (2), Art 5.

$$\frac{a'}{\sqrt{3}}=\frac{b'}{1}=\frac{c'}{\tan\alpha};$$

$$\therefore\ \tan\alpha_x=\sqrt{3},$$

$$\tan\alpha_y=\frac{1}{\tan\alpha}=\cot\alpha,$$

$$\tan\alpha_z=\frac{1}{\sqrt{3}}\tan\alpha\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ \quad(a)$$

$$\therefore\ \alpha_x=60^\circ\ ;\ \alpha_y=\frac{\pi}{2}-\alpha.$$

The other angular element α_z is given by equation (a.)

Hence one of the angular elements is always equal to 60°.

15. The results of this investigation may therefore be summed up by the following statement:—All crystals included in the rhombohedral system are merely the particular forms which crystals of the prismatic system assume when one angular element is equal to 60°.

16. Tables are given in the paper containing the symbols of the forms of the prismatic system corresponding to the forms observed in the various minerals belonging to the rhombohedral system.

The following is an example selected from the tables:—

Apatite.

Angular elements—

100, 110 = 60°; 010, 011 = 34° 20′; 001, 101 = 40° 13′.

Simple forms—

100, 010, 001, 011, 101, 110, 111, 012, 021, 102, 201, 310, 112, 114, 311, 312, 314.

The following forms are always hemihedral with parallel faces—

210, 130, 320, 350, 510, 910, 121, 122, 532, 534, 712, 714, 615, 3 7 10, 9 5 10.

Cleavage—

100, 001, 110.

001 not so easily obtained as 100, 110. The angles between the faces are of course the same whatever be the symbols used to represent them. For Apatite they are, therefore, those given on p. 485 of Miller's "Mineralogy."

17. The angular elements of Calcite are—

100, 110 = 60°; 010, 011 = 45° 23′·4; 001, 101 = 29° 40′.

Those of Aragonite are—

100, 110 = 58° 5′; 010, 011 = 40° 50′; 001, 101 = 35° 47′.

These angular elements differ considerably. Aragonite and Calcite will therefore continue to form distinct mineral species.

18. The author hopes soon to be able to publish the results of investigations which appear to show that the crystals included in the oblique and anorthic systems are formed by the combination of hemihedral and tetartohedral forms of the prismatic system.

8. Preliminary Note on the Connection between the Form and Optical Properties of Crystals. By Alfred R. Catton, B.A., F.C.P.S., Scholar of St John's College, Cambridge.

1. It is the object of this note to give an account of the results of investigations, which have had for their object the discovery of the connection between the form and optical properties of crystals.

It is believed that in the results here given, some of the principal difficulties of this important problem have been overcome.

2. The first step towards the solution of this problem was made by Sir David Brewster in 1818. He discovered that crystals belonging to the prismatic, oblique, and anorthic systems, are biaxal; those belonging to the pyramidal and rhombohedral systems uniaxal, while crystals of the cubic system do not possess double refraction (a fact which had been previously stated by Hauy).

In the prismatic system, the optic axes always lie in a plane containing two of the crystallographic axes, and make equal angles with each of these axes.

Now the theoretical investigations of Fresnel have shown that the optic axes lie in a plane containing the axes of greatest and least elasticity, and make equal angles with each of these axes.

Hence it follows, that, in the prismatic system, the axes of elasticity coincide with the crystallographic axes.

In the oblique system, one axis of elasticity coincides with that crystallographic axis which is at right angles to each of the other two.

In the anorthic system, the position of the axes of elasticity is subject to no known law.

3. These facts show that there is a fundamental connection between crystalline form and optical properties. Of late, nevertheless, many eminent physicists, following the views of De Senarmont, have doubted the existence of such a connection. We shall, therefore, show hereafter that the experiments of De Senarmont as well as those of Des Cloizeaux are not at all opposed to, but important confirmations of, the views here adopted, which are founded on the facts stated above, and in addition on the remarkable relations that have been observed between hemihedral forms and rotatory polarisation in quartz, and dextro- and lævo- tartaric acids.

4. In this note we shall consider in detail the connection between the form and optical properties of crystals belonging to the prismatic system. For crystals belonging to the oblique and anorthic systems, the investigation is still in progress.

5. We proceed to consider the following problem.

"Given the angular elements of a crystal belonging to the prismatic system, to find the angle between the optic axes."

We have before observed, that, in the prismatic system, the optic axes always lie in a plane containing two of the crystallographic axes, and make equal angles with each of these axes.

First, let the optic axes lie in the plane containing the parameters c and a, and each make an angle ω_c with the axis c. Our object is to find an expression for some function of ω_c, say $\tan \omega_c$, in terms of the parameters a, b, c.

Now crystals belonging to the pyramidal system have only one optic axis, which coincides with the axis c, if $a=b$. Hence, when $a = b$, the angle which each optic axis makes with the axis c vanishes; or, in other words, both optic axes coincide with the axis c.

Hence, ω_c and $\therefore \tan \omega_c$ vanishes when $a = b$.

$\therefore \tan \omega_c$ is divisible by $a - b$.

6. In an abstract of a paper on the "Rhombohedral System," published in the present number of the "Proceedings," it is shown that the crystals hitherto included in the rhombohedral system are merely the particular forms which crystals of the prismatic system assume when one angular element is equal to 60°.

But crystals belonging to the rhombohedral system have only one optic axis, and this, as is shown in the paper referred to, coincides with the axis c if the angular element 100, 110 = 60° ; *i.e.*, if $\frac{a}{b} = \sqrt{3}$.

Hence when $a = b\sqrt{3}$, the angle which each optic axis makes with the axis c vanishes.

Hence, ω_c and $\therefore$ tan ω_c vanishes when $a = b\sqrt{3}$.

$$\therefore \tan \omega_c \text{ is divisible by } a - b\sqrt{3}.$$

But we have before shown that it is divisible by $a - b$.

$$\therefore \tan \omega_c \text{ is divisible by } (a-b)(a-b\sqrt{3}).$$

But all uniaxal crystals belong either to the pyramidal or rhombohedral systems.

Hence these are the only factors containing a and b, by which tan ω_c is divisible.

7. Again, if $\omega_c = \frac{\pi}{2}$, the optic axes coincide with the axis a.

But when this is the case, by the same reasoning as before, either $b = c$, or $b = c\sqrt{3}$; $\therefore \omega_c$ must $= \frac{\pi}{2}$, or tan ω_c must become infinite when $b - c = o$, or $b - c\sqrt{3} = o$.

Hence the expression for tan ω_c, in terms of a, b, c, must be a fraction, the denominator of which contains the factors $b - c$ and $b - c\sqrt{3}$; and, for the same reason as before, these are the only factors in the denominator containing b and c.

Hence, since tan ω_c is of no dimensions, the expression for tan ω_c in terms of a, b, c, must be of the form

$$\tan \omega_c = \frac{\mathrm{C}\,(a-b)^m(a-b\sqrt{3})^n}{\mathrm{A}\,(b-c)^m(b-c\sqrt{3})^n}$$

where C is independent of a and b, and A of b and c.

8. Crystals belonging to the cubic system do not possess double refraction ; in other words, they have an infinite number of optic axes.

Hence, tan ω_c ought to become indeterminate, when $a - b = o$ and $b - c = o$, which is the case with the above expression.

9. We have shown from physical considerations, that the factors $a - b$, and $a - b\sqrt{3}$, must enter into the expression, for tan ω_c at least, to the first degree ; there is, however, no physical reason for

supposing that these factors enter to a degree higher than the first, and we are therefore led to assume that they enter to the first degree only. Hence the expression for tan ω_c becomes

$$\tan \omega_c = \frac{\mathrm{C}\ (a-b)\,(a-b\sqrt{3})}{\mathrm{A}\ (b-c)\,(b-c\sqrt{3})} \quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \quad (1.)$$

Similarly, if the optic axes lie in the plane containing the axes a and b, and if ω_a be the angle which each makes with the axis a,

$$\tan \omega_a = \frac{\mathrm{A}(b-c)\,(b-c\sqrt{3})}{\mathrm{B}(c-a)\,(c-a\sqrt{3})} \quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \quad (2.)$$

Also, if the optic axes lie in the plane containing the axes b and c, and if ω_b be the angle which each makes with the axis b,

$$\tan \omega_b = \frac{\mathrm{B}(c-a)\,(c-a\sqrt{3})}{\mathrm{C}(a-b)\,(a-b\sqrt{3})} \quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \quad (3.)$$

We shall show, by the comparison of these formulæ with observation, that $\frac{\mathrm{A}}{a} = \frac{\mathrm{B}}{b} = \frac{\mathrm{C}}{c}$.

10. These formulæ can be easily expressed in terms of the angular elements α_1, α_2, α_3.

$$\text{For } \tan \alpha_1 = \frac{a}{b} \qquad \tan \alpha_2 = \frac{b}{c} \qquad \tan \alpha_3 = \frac{c}{a}$$

Hence, putting $\frac{\mathrm{A}}{\mathrm{B}} = \frac{a}{b}$, the formula (2.) may be written in the form,

$$\tan \omega_a = \frac{(\cot \alpha_2 - \cot 45^\circ)\,(\tan \alpha_2 - \tan 60^\circ)}{(\cot \alpha_3 - \cot 45^\circ)\,(\tan \alpha_3 - \tan 60^\circ)}$$

and so for the others.

11. The angular elements of a crystal are to a certain extent arbitrary; thus the parameters may be changed from a, b, c, to pa, qb, rc—where p, q, r are positive integers none of which are zero—provided the symbols of the simple forms are altered accordingly.

With the new parameters formula (2.) becomes

$$\tan \omega_a = \frac{pa}{qb}\,\frac{(qb-rc)\ (qb-rc\sqrt{3})}{(rc-pa)\ (rc-pa\sqrt{3})}$$

We conclude, therefore, that finite and integral values of p, q, r may be found such that, with the angular elements given in

Miller's "Mineralogy," or elsewhere, the calculated agrees with the observed angle between the optic axes.

We proceed to the comparison of these formulæ with observation.

12. *Chrysoberyl.*—In this mineral, the optic axes lie in the plane bc, and make angles of 13° 55′, with axis c.

Let ω be this angle.

$$\text{Then } \tan\omega = \cot\omega_b = \frac{C\left(1 - \frac{b}{a}\right)\left(1 - \frac{b}{a}\sqrt{3}\right)}{B\left(\frac{c}{a} - 1\right)\left(\frac{c}{a} - \sqrt{3}\right)} \text{ by formula (3.)}$$

Angular elements,—

$$100,\ 110 = 64° \ 49'; \quad 010,\ 011 = 39° \ 1'; \quad 001,\ 101 = 30° \ 7'.$$

$$\therefore \ \frac{b}{a} = \cot 64° \ 49' = \cdot 4702.$$

$$\frac{c}{a} = \tan 30° \ \ 7' = \cdot 5801\cdot$$

$$\therefore \ \tan\omega = \frac{\cdot 5298 \times \cdot 1856}{\cdot 4199 \times 1\cdot 152} \times \frac{C}{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a.)$$

$$= \cdot 20326 \text{ if } \frac{C}{B} = 1.$$

On this supposition, ω is somewhat less than 11° 30′, which differs by 2° 25′ from the observed value of 13° 55′; this difference is too great to be ascribed either to errors in the determination of the angular elements, or of the angle between the optic axes. Hence $\frac{C}{B}$ is not equal to unity.

The value of $\frac{C}{B}$ must be such that the value of ω given by equation (a) is equal, or nearly so, to 13° 55′.

We find this to be the case on trial, if $\frac{C}{B} = \frac{c}{b}$.

For chrysoberyl $\frac{c}{b} = \cot 39° \ 1' = 1\cdot 2337$.

Hence, equation (a) becomes

$$\tan\omega = \cdot 20326 \times 1\cdot 2337.$$

$$= \cdot 2507.$$

$$\therefore \ \omega = 14° \ 4'\cdot 5;$$

which only differs from 13° 55′ by 9′·5, a difference quite within the limits of errors of observation.

Hence, from the optical properties of chrysoberyl, we deduce that, if the optic axes lie in the plane bc,

$$\tan \omega_b = \pm \frac{b\,(c-a)\,(c-a\sqrt{3})}{c\,(a-b)\,(a-b\sqrt{3})}.$$

The sign $\pm$ is used, for in the reasoning by which the above formula has been deduced, there is nothing to show whether the factor corresponding to $a=c$ is $a-c$, or $c-a$, or that corresponding to $c=a\sqrt{3}$, $c-a\sqrt{3}$, or $a\sqrt{3}-c$, &c.

13. *Aragonite*—

In this mineral the optic axes lie in the plane ca, and for the fixed line B make angles of 9° 2′ 41″·5 with the axis c, and of 9° 20′ 20″ for the fixed line H.*

Hence by formula (1.),

$$\tan \omega_c = \frac{(pa-qb)\,(pa-qb\sqrt{3})}{(qb-rc)\,(qb-rc\sqrt{3})}\,\frac{rc}{pa}$$

Angular elements—

100, 110 = 58° 5′; 010, 011 = 40° 50′; 001, 101 = 35° 47′.

$$\therefore\ \frac{a}{b}=1{\cdot}6055 \quad \frac{c}{b}=1{\cdot}1571 \quad \frac{c}{a}={\cdot}72078.$$

In the above formula put $p=q=1$ and $r=2$.

The form denoted in Miller's "Mineralogy" by the symbol 201 is thus taken as the form 101.

$$\text{Hence } \tan \omega_c = \frac{{\cdot}6055 \times {\cdot}1266}{2{\cdot}3142 \times 3{\cdot}0084} \times 2 \times {\cdot}72078$$

$$= {\cdot}110106 \times 2 \times {\cdot}72078$$

$$= {\cdot}15872$$

$$\therefore\ \omega_c = 9^\circ\ 2' \text{ nearly,}$$

which differs from Kirchhoff's determination for mean rays, viz., 9° 9′ 35″ by 7′ 35″.

14. *Karstenite* (Anhydrite).—Optic axes in plane ab, and make angles of 20° 18′·5, with axis b. Here,

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* Kirchhoff (*Pogg. Annalen.* cviii. (1859), p. 574).

$$\tan\omega = +\frac{(c-a)(c-a\sqrt{3})}{(b-c)(b-c\sqrt{3})}\,\frac{b}{a}$$

$$\text{putting } \frac{B}{A}=\frac{b}{a}.$$

Angular elements,—

010, 011 = 42° 17′; 001, 101 = 44° 25′.

If these be altered thus,—

010, 011 = 42° 30′ (diff. of 13′); 001, 101 = 44° (diff. of 25′),

we have $\tan\omega = \cdot 37134$

$\therefore\ \omega = 20°\ 30'$ nearly, which differs by 11′·5 from the observed value.

15. *Nitre.*—Optic axes in plane *ca*, and make angles of about 3° with axis *c*.

Angular elements,—

100, 110 = 59° 25′; 010, 011 = 40° 8′.

If these be altered thus,—

100, 110 = 59° 40′ (diff. of 15′); 010, 011 = 40° (diff. of 8′).

The value of ω given by the formula,

$$\tan\omega = \pm\frac{(a-b)(a-b\sqrt{3})}{(b-c)(b-c\sqrt{3})}\,\frac{c}{a}$$

is 3° 20′ nearly, differing by about 20′ from the observed angle.

16. *Cerussite.*—In this mineral the optic axes lie in the plane *bc*, and make angles of 4° 8′ with axis *c*.

Angular elements,—

100, 110 = 58° 36′·5; 010, 011 = 40° 9′·5; 001, 101 = 35° 52′.

$$\therefore\ \frac{b}{a}=\cdot 6102 \qquad \frac{c}{a}=\cdot 723 \qquad \frac{c}{b}=1\cdot 1851.$$

Hence equation (3.) becomes

$$\tan\omega = \frac{\cdot 3898\times\cdot 0569}{\cdot 277\times 1\cdot 0091}\times\frac{C}{B}$$

$$=\cdot 08 \text{ if } \frac{C}{B}=1.$$

$\therefore\ \omega = 4°\ 36'$ nearly, which differs by 28′ from the observed value.

If $\frac{C}{B}=\frac{c}{b}=1\cdot 1851$, the equation becomes,

$$\tan\omega = \cdot 08\times 1\cdot 1851 = \cdot 0948.$$

$\therefore\ \omega = 5°\ 24'$, which differs from the observed value by 1° 16′.

17. By making small alterations in the angular elements of Glaserite, Brookite, Epsomite, Goslarite, Cordierite, the calculated may be made to agree with the observed angle within the limits of errors of observation.

18. Hence if $\frac{A}{a}=\frac{B}{b}=\frac{C}{c}$, the expressions (1), (2), (3), for the inclination of the optic axes to the crystallographic axes *c*, *a*, *b*, when they lie in the planes *ca*, *ab*, *bc*, respectively, agree with observation, in some cases, without making any, or only slight changes, in the angular elements; but in other cases it is necessary to make considerable changes. It is therefore important to consider whether the changes required are greater than the possible errors in the angular elements.

19. Now, the measurements of the angles between the faces of different specimens of the same mineral, by the same or by different observers, often vary very considerably, sometimes as much as 1° or 2°. In Karstenite, for instance, the angle between the faces of the form 110, which are truncated by the face 010, is variously stated as 96° 36′ or 98° 54′, which differ by 2° 18′. This difference is too great to be ascribed to errors of observation. It only appears possible to account for these differences in the measurements of the same or of equally accurate observers, by supposing that the angular elements of a given mineral are liable to vary slightly in different specimens. The question is, to what causes is this variation due? Now, in isomorphous substances, the angular elements often differ by a degree or more. For instance, the angular elements of the minerals isomorphous with Aragonite are,—

	100, 100	010, 011
Aragonite, ...	58° 5′	40° 50′
Cerussite, ...	58°36′·5	40° 9′·5
Strontianite, . . .	58° 39′	40° 5′
Witherite, ...	59° 15′	38° 45′
Alstonite, ...	59° 25′·5	38° 39′

It will be observed that the angular elements of Alstonite differ respectively by 1° 20′·5 and 2° 11′ from those of Aragonite, those of the other minerals differing by less. Now Alstonite may be considered as Aragonite, in which half the carbonate of lime is replaced by carbonate of baryta. Hence we may say, that when half the lime in Aragonite is replaced by baryta, the angular elements of the latter are altered from 58° 5′ and 40° 50′ to 59° 25′·5, and 38° 39′ respectively.

Reasoning of a similar kind may be applied in a number of other cases.

Hence, we conclude that the angular elements of a mineral vary when the normal constituents are replaced by isomorphous substances.

20. Again, all crystals, except those of the cubic system, expand, in general, differently in different directions under the action of heat. This is always the case in crystals belonging to the prismatic, oblique, and anorthic systems.

Hence a change of temperature alters the angles between the faces, and therefore the angular elements, of crystals belonging to these systems, and the angular elements will in general be permanently altered if the temperature be sufficiently raised.

Hence the angular elements of a mineral depend upon the temperature to which it is or has been exposed.

21. We conclude, therefore, that the angular elements, instead of being constant in all specimens of the same mineral, vary within narrow limits, according to the amount of isomorphous replacement and the temperature to which it is or has been exposed;* and further, that the amount of this variation is quite as great as the differences which must be made in the angular elements, given in Miller's "Mineralogy," in order to make the calculated agree with the observed angle between the optic axes. It is possible that, in a number of cases, the angular elements of the crystal employed

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* Many apparent cases of dimorphism may possibly be hence explained. Thus, Aragonite is not pure carbonate of lime, but is always associated with variable quantities of the carbonates of strontia, lead, or manganese. Calcite sometimes contains a considerable quantity of foreign substances,—such as protocarbonate of iron, oxide of zinc, &c. Hence, if isomorphous replacement produces changes in the angular elements, these differences in the composition of Calcite and Aragonite would be sufficient to account for the differences in their angular elements (supposing them both referred to the *prismatic* system). Graphite sometimes contains as much as 10 per cent. of iron; titanic acid is always associated in anatase, rutile, and brookite, with variable quantities of sesquioxide of iron. Differences in the angular elements may also be due to differences in the temperature at which the crystals were formed.

for determining the angle between the optic axes, were, from the causes mentioned, different from those given by Professor Miller.

22. But if the angular elements of a mineral vary within certain limits, the angle between the optic axes, calculated by the formulæ given in this note, must also vary within corresponding limits.*

The causes, therefore, which produce in a given mineral the variations in the angle between the optic axes are the same as those which produce the variations in the angular elements. That the angle between the optic axes of a crystal varies by isomorphous replacement, is established by the experiments of De Senarmont;† that it varies with the temperature to which the crystals are or have been exposed, is established by those of Des Cloizeaux.‡

The existence of this variation, therefore, far from being opposed to the existence of a connection between the form and optical properties of crystals, as argued by De Senarmont,§ is strongly in support of it.

23. Suppose that from either of the causes mentioned, the parameters a, b, c, become $a+x$, $b+y$, $c+z$, where x, y, z, are small quantities. Let the optic axes lie in the plane ca, and make an angle ω with the axis c, and let ω' be the angle corresponding to the new parameters. It is easily seen that

$$\tan\omega' = \tan\omega\left[1+\frac{\dfrac{x-y}{a-b}-\dfrac{y-z}{b-c}+\dfrac{x-y\sqrt{3}}{a-b\sqrt{3}}-\dfrac{y-z\sqrt{3}}{b-c\sqrt{3}}}{1+\dfrac{y-z}{b-c}+\dfrac{y-z\sqrt{3}}{b-c\sqrt{3}}}\right]$$

neglecting $\dfrac{c+z}{a+x}$

Hence, if 100, 110 be nearly 45° or 60°, and $x-y$ not very small in the former case, nor $x-y\sqrt{3}$ in the latter, while 010, 011 differs considerably from 45° or 60°, $\tan\omega' - \tan\omega$ will be large. In this case, therefore, the angle between the optic axes will vary considerably.

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* Topaz and mica are well known examples of this variation of the angle between the optic axes.

† Annales de Chemie, third series, xxxiii. p. 391.

‡ Annales des Mines, sixth series, ii. p. 327.

§ Ibid., p. 433.

If 010, 011 be nearly 45° or 60°, and $y-z$ not very small in the former case, nor $y-z\sqrt{3}$ in the latter, while 100, 110 differs considerably from 45° or 60°, the value of tan ω' − tan ω, though considerable, will not be so great as before.

If 100, 010 and 010, 011 are both nearly equal to 45° or 60°, tan ω' will differ very little from tan ω, except when $x-y$ and $y-z$, or $x-y\sqrt{3}$ and $y-z\sqrt{3}$ differ considerably.

Hence are explained the variations of the angle between the optic axes produced by an increase of temperature in Brookite and Chrysoberyl, as observed by Des Cloizeaux.

24. We find that the expression for tan ω_c in terms of a, b, c, contains the factor $\frac{c}{a}$. For the existence of this factor no *à priori* physical reason can be assigned. Now $c=o$ represents an infinite plane, containing the crystallographic axes a and b, and tan ω_c vanishes when $c=o$. The physical interpretation of this factor therefore is, that such an infinite plane has only one optic axis which coincides with the axis c. We may compare this result with the fact, that if a ray of light is incident on a crystal of the prismatic system, in the direction of one of the principal planes, one of the refracted rays, which is polarised in that principal plane, follows the ordinary law of refraction.

25. Again, the expression for tan ω_c becomes indeterminate if

$$a-b=o, \text{ and } b-c\sqrt{3}=o;$$
$$\text{or, } b-c=o, \text{ and } a-b\sqrt{3}=o;$$
$$\text{or, } a-b\sqrt{3}=o; \text{ and } b-c\sqrt{3}=o.$$

Hence, if the angular element of a crystal of the pyramidal system were equal to 60°, it would not possess double refraction; the same would be the case if one of the angular elements of a crystal of the rhombohedral system were equal to 60°.

26. If a', b', c' be the optical constants in descending order of magnitude, and ω the angle which each optic axis makes with the axis of least elasticity, we have,

$$\tan^2\omega=\frac{a'^2-b'^2}{b'^2-c'^2},$$

assuming Fresnel's expression for the angle between the optic axes in terms of the optical constants;* but if the optic axes lie in the plane ca, we have also,

$$\tan{}^2\omega = \frac{(a-b)^2\,(a-b\sqrt{3})^2c^2}{(b-c)^2\,(b-c\sqrt{3})^2a^2}\,;$$

$$\therefore\ \frac{a'^2-b'^2}{(a-b)^2(a-b\sqrt{3})^2c^2} = \frac{b'^2-c'^2}{(b-c)^2\,(b-c\sqrt{3})^2a^2},$$

which is a relation between the optical and crystallographic constants.

Similarly, if the optic axes lie in the plane ab, and $b'c'a'$ be in descending order of magnitude,

$$\frac{b'^2-c'^2}{(b-c)^2\,(b-c\sqrt{3})^2a^2} = \frac{c'^2-a'^2}{(c-a)^2\,(c-a\sqrt{3})^2b^2},$$

and if the optic axes lie in the plane bc, and $c'a'b'$ be in descending order of magnitude,

$$\frac{c'^2-a'^2}{(c-a)^2\,(c-a\sqrt{3})^2b^2} = \frac{a'^2-b'^2}{(a-b)^2\,(a-b\sqrt{3})^2c^2}.$$

27. A serious objection may be made to formulæ such as the above, expressing relations between a', b', c', and a, b, c.

In the formula

$$\frac{a'^2-b'^2}{b'^2-c'^2} = \frac{(a-b)^2\,(a-b\sqrt{3})^2c^2}{(b-c)^2\,(b-c\sqrt{3})^2a^2}$$

for instance, the left side of the equation is a function of the wave-length.

But since a, b, c depend only on the angles between the faces of crystals, which are of course invariable, the right side of the equation is independent of the wave length.

The only way of overcoming this difficulty appears to be by supposing that the apparent angles between the faces of crystals of the prismatic system, as determined by the reflective goniometer, may vary with the kind of light employed. Some experiments of Sir David Brewster on crystalline reflexion appear to countenance such a supposition; but the problem of crystalline reflexion is involved in obscurity.

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* The measurements of Kirchhoff (*Ibid.*) have shown that this formula agrees closely with observation in the case of Aragonite.

Also, if $a'=b'$ the crystal becomes uniaxal, and the optic axes coincide with axis c'; hence $\tan{}^2\omega$ ought to vanish when $a'=b'$.

If $b'=c'$ the optic axes coincide with axis a'; hence $\tan{}^2\omega$ ought to become infinite when $b'=c'$. Fresnel's expression fulfils these conditions.

28. Since all the properties of crystals, as the values of the principal refractive indices, the wave velocities, the form of the wave surface, &c., depend on the values of a', b', c', it follows that the complete solution of the problem of the connection between the form and optical properties of crystals consists in determining a', b, c' as functions of a, b, c.

$$\text{Let } \frac{a'^2}{f_1abc} = \frac{b'^2}{f_2abc} = \frac{c'^2}{f_3abc}$$

where a' b' c' are in descending order of magnitude,

$$\text{then} \quad f_1abc - f_2abc = (a-b)^2 (a-b\sqrt{3})^2 c^2$$

$$f_2abc - f_3abc = (b-c)^2 (b-c\sqrt{3})^2 a^2$$

$$\therefore \frac{a'^2}{(a-b)^2(a-b\sqrt{3})^2c^2 + (b-c)^2(b-c\sqrt{3})^2a^2 + f_3abc}$$

$$= \frac{b'^2}{(b-c)^2(b-c\sqrt{3})^2a^2 + f_3abc}$$

$$= \frac{c'^2}{f_3abc}$$

Hence only one function f_3abc remains to be determined. Symmetry would suggest the value $(c-a)^2 (c-a\sqrt{3})^2 b^2$ for f_3abc. We have not as yet, however, attempted to determine the value of f_3abc by the comparison of this formula with observation.

29. The investigation of the optical properties of crystals belonging to the oblique system is still in progress. The following is the method employed in this investigation. Each crystal is referred to three rectangular axes, one axis being perpendicular to the plane of the optic axes, the other two being the internal and external bisectors of the angle between the optic axes. The new parameters are calculated by means of formulæ investigated in a paper on the "Rhombohedral System;" and thence the angle between the optic axes is found, as if the crystal belonged to the prismatic system, by means of the formulæ given in this note. The angle between the optic axes of one mineral belonging to the oblique system has been calculated by this process; and the calculated has been found

to agree approximately with the observed angle. If this should prove to be generally the case, it will not only be a solution of the problem which forms the subject of this note for crystals of the oblique system, but it will prove that these crystals are formed according to the same laws of symmetry as crystals of the prismatic system.

9. A Contribution to the History of the Oxides of Manganese. By W. Dittmar, Esq.

It is known that oxides of manganese, when ignited in air, either absorb or lose oxygen, until they attain the composition expressed by the formula $Mn_3\,O_4$. Schneider found that when they are heated in pure oxygen, they leave the oxide $Mn_2\,O_3$.

This latter result is not what one might have expected, for we know of no positive action exerted on manganese oxides or on pure oxygen by nitrogen. Thus air should act like dilute oxygen, producing the same oxide.

To explain Schneider's result, we must suppose either that artificial oxygen differs from that contained in air, or, what is more likely, that the amount of oxygen which any given quantity of manganese can take up when heated in it, depends not only on the temperature, but also *on the tension of the oxygen* employed.

If, then, it is true, that at a red heat, in presence of oxygen of the tension of one atmosphere, $Mn_2\,O_3$ is a stable compound, while the same oxide, exposed to the same temperature in presence of oxygen of the tension of one-fifth of an atmosphere, is reduced to $Mn_3\,O_4$, the question naturally arises, What oxides will be formed in presence of oxygen of different tensions from those two above mentioned? Is there a continuous series of oxides corresponding to a continuous series of oxygen-tensions under which they have been formed?

These questions seemed to me to merit investigation, especially as their results would form an addition to our very imperfect knowledge of the influence of physical conditions on chemical reactions.

In the first series of experiments, pure peroxide of manganese, in a porcelain or platinum boat, was placed in a porcelain tube, kept at a bright red heat, while a current of nitrogen or oxygen, or of a mixture of these two gases, was passed over it. The resulting oxide was weighed and analysed by determining the quantity of protoxide it yielded when heated in a current of hydrogen. Most of the experiments were carried on under ordinary atmospheric pressure; but in a few cases, the tension of the gas was made less than one atmosphere by connecting the exit end of the porcelain tube with a large bell-jar, within which the pressure was diminished, and kept constant by means of an air-pump.

Eighteen experiments gave the following results.

In all cases, either black $Mn_2 O_3$, or brown $Mn_3 O_4$ was obtained; *no intermediate oxides were formed.*

In all the cases in which $Mn_3 O_4$ was obtained, the partial tension of the oxygen lay between 0 and 0·21 atmospheres (the latter tension being that of the oxygen of the atmosphere). In all the experiments in which $Mn_2 O_3$ was obtained, the partial tension of the oxygen lay between 0·25 and one atmosphere.

Oxygen tensions intermediate between 0·21 and 0·25 did not occur in the series; but in spite of this gap, the results obtained render it highly probable that the function expressing the relation between the composition of an oxide of manganese which is formed at a red heat in an atmosphere of oxygen, and the tension of that oxygen is discontinuous, so that $Mn_2 O_3$ is formed whenever the tension *exceeds*, and $Mn_3 O_4$ whenever it is *below a certain definite limit.*

To determine whether such a point of discontinuity really exists, and if so, its position, a second series of experiments was performed, which were made as uniform, and therefore comparable, as possible. In all cases, the $Mn O_2$ was heated in mixtures of nitrogen and oxygen of exactly known composition, kept at, or very near to, the tension of the surrounding atmosphere. The percentages of oxygen in those mixtures were so chosen as gradually to inclose the value sought for within narrower and narrower limits In each experiment, the height of the barometer was determined, in order to know exactly the partial tension of the oxygen in the mixture.

Eleven experiments were made in all. In eight, the oxides obtained were either pure $Mn_2 O_3$, or $Mn_3 O_4$. In three, the oxides were obtained together, not mixed, but as it were side by side, and occupying different parts of the boat. In those cases, in which

Mn_3O_4 was obtained alone, or at least in which it predominated, the partial tension of the oxygen (when measured by the height of the equivalent column of mercury) did not exceed 7·07 inches.

In those cases in which Mn_2O_3 was obtained, or at least predominated, the partial tension of the oxygen was equal to, or greater than, 6·9 inches. Thus the point of discontinuity lies between the two limits 6·9 and 7·07 inches.

Within these two tensions, sometimes the one and sometimes the other oxide was obtained. This fact does not speak, however, against the existence of some definite point of discontinuity, as the composition of the resulting oxide depends on the temperature at which the experiment is performed, as well as the tension of the oxygen employed. A diminution of temperature is probably of the same effect as an increase in the tension of the oxygen, and *vice versa.* Thus, if the tension of the oxygen in a given mixture of that gas and nitrogen is just equal to that corresponding to the point of discontinuity, for a definite temperature t, a temperature slightly above that will cause the formation of Mn_3O_4, and one slightly below will produce Mn_2O_3.

In one experiment I succeeded in producing the two oxides by means of the same mixture of nitrogen and oxygen, only the tension being slightly varied. Mn_2O_3, heated in this mixture at the atmospheric pressure of 29·9 inches, remained unchanged, while at the tension of 29·5 it was reduced to Mn_3O_4. This is quite within the limit of ordinary barometrical variations.

10. Notice of Glacial Clay, with Arctic Shells, near Errol, on the Tay. By the Rev. Thomas Brown, F.R.S.E.

The author referred to the paper on the Elie Glacial Clay, with Shells, read by him on the 2d of March 1863. During the succeeding summer he learned from Dr M'Bain, that shells had been found in a brickfield near Errol, but so badly preserved, that none of the species had been determined. During the autumn, while staying in the neighbourhood of Perth, he took occasion to visit the Errol brickfield, and found that the shells, which occur in considerable abundance, are precisely the same group with those at Elie. The *Leda truncata,* for example, which is the characteristic shell of the deposit at Elie, is yet more abundant at Errol, along with the *Pecten Grœnlandicus*, and most of the peculiar Arctic shells of the deposit. Some interesting additions to the list of Arctic species were obtained at Errol. The skeleton of a seal was also found; but the description of these fossils, as well as the observations made on the nature of the deposit and its stratigraphical relations, were reserved for a future occasion. The author merely wished to direct attention to the fact, that those glacial clay beds of the Forth are found also in the Tay.

11. Notes on the Boulder-Clay at Greenock and Port-Glasgow. By the Rev. R. Boog Watson, B.A., F.R.S.E., Hon. Mem. Nat. Ver. Lüneburg.

In Greenock, excavations have lately been made for a new gasometer. The works are now completed, but the superintendent, a most intelligent man, took me to the place, and told me what they had found in the course of digging.

The site of the excavation is close to the shore, and very little above the tide-mark.

At the south-east corner of the works, *i.e.*, most remote from the sea, the workmen reached the rock at a depth of 20 feet. It was a soft shale, and I could not ascertain that any striations were observed on it. Probably there were none preserved on such material. Its upper surface was flat. Towards the sea, or north and west, the rock sloped downwards very steeply, and this seaward face was covered by a great bed or bank of sand, that sloped up from the edge of the rock, and as it receded rose higher as a bank.

The flat surface of the rock was covered by from 6 to 12 inches of a fine soft clay without stones. I could not learn whether this clay overlay or dipped under the sand-bank, or whether it simply disappeared altogether at the edge of the rock, where the sand-bank began. The last alternative seems the most probable.

Above the clay lay from 18 to 20 feet of boulder-clay, a little thinner, of course, over the back of the sand-bank. The boulder-clay was dark in colour, sandy, and full of striated stones, some of which were of considerable size—in short, exactly similar to what we have at Leith and Newhaven.

The interest of this section lies in the fact of its proving, that the boulder-clay in this case was deposited in water. This is obvious, not only from the sedimentary character of the underlying clay and sand, with which the boulder-clay is here associated, but also from the soft nature and exposed condition of these underlying beds which could not possibly have resisted the pressure of ice. The boulder-clay seems here, therefore, to have been dropped from under the edge of the ice-cake covering of our land just where it began to float at the shore line, and thus quietly to have buried the true marine deposits now underlying it.

Another locality, where I recently examined the boulder-clay, is in the cutting of the New Wemyss Bay Railway, where it approaches its junction with the present Greenock Railway, just below Port-Glasgow. The new railway cutting here runs at a height of 60 feet above the sea, along the edge of the steep slope, the base of which is the flat known as the 40-foot sea-beach. The old railway runs here along this flat; and it was in this neighbourhood that the shell-bed described by Mr Smith of Jordanhill occurs in a bed of sand overlying the boulder-clay. The boulder-clay is somewhat redder, but otherwise exactly like that in the railway cutting which is now being made between Newhaven and Leith. It is dark, sandy, and full of striated stones. Through it there runs horizontally a well marked stratum of fine sand and clay about two inches thick, which I traced for several yards, till it was lost in the debris of the cutting In the midst of this boulder-clay at a height of about 60 feet above the sea, I found several fragments of shells, one or two of which seemed to belong to an *astarte;* the others were unrecognisable. Under the boulder-clay the rock has been laid bare, and both its seaward face and upper surface are well seen. It is a coarse sandstone; its upper surface is very strongly striated and well rounded.

Here then, as in the former case, we have evidence that the boulder-clay was deposited in the sea. It was land ice which formed the materials of the boulder-clay, and consolidated them by its pressure. This is now generally admitted, but the presence of the shells and the existence of the stratum of sand and clay indicate that these materials were deposited on a surface covered by the sea. But there are indications here of a still more interesting fact. The steep bank or cliff which pretty generally along our coasts rises from a flat elevated about 40 feet above the sea, has always been spoken of as a beach formed by the sea, and not a few calculations have been made to determine from a comparison with our existing sea-beach the length of time during which the sea must have stood at the higher level, in order to its having eroded our land in the very marked way which the upper or 40-foot terrace presents. But plainly, before any such calculations are legitimate, it ought to be proved that the 40-foot terrace is really the work of the sea. Now, this seems to have been somewhat hastily assumed. Undeniably, the sea does act powerfully on the shore, cutting out a step or forming a beach line along its margin; further, it is obvious, that that margin was at one time 40 feet higher along our coasts than now, and the inference has been jumped at, that the terrace which does certainly exist at that level was formed by the sea. In other words, a connection of proximity between the sea and the 40-foot terrace being proved, the connection of cause and effect has been assumed. But is the assumption legitimate? Now this cutting at Port-Glasgow shows that the 40-foot terrace exists not merely on the external surface, but in the rock beneath. The superficial deposits are merely the clothing of a rocky skeleton beneath, and the terrace which we can trace on the surface, we find existing in the rock below. What then gave its form to that rocky skeleton below? Was the sea the agent which cut the terrace there? Plainly not, for when the sea beat on the shore 40 feet above its present level, the rock, in its present form, was buried even deeper than now under that mass of boulder-clay which still covers it, and which the sea, at this particular spot at least, and at many others, was not able to penetrate. But was the boulder-clay already there? Admitting what is obvious, that *if there*, it alone and not the underlying rock could be fashioned by the sea, are we sure that it really was already deposited, or may it not rather have been deposited at the very time we are speaking of, when the land in its upward progress still stood 40 feet lower than now?

Now, of this earlier presence of the boulder-clay we have abundant proof. The rock surface here and elsewhere on our coasts is sharply striated. The striating agent, it is now generally admitted, was land ice in the form of a thick glacier cake, grinding downwards, and pressing outwards towards the sea. This glacier cake must have been slowly floated off by the gradual subsidence of the land. As it floated off the boulder-clay was deposited under the edge of the ice-foot on the freshly striated surface of the rock, protecting it at once from the action of the surf, consolidated under the pressure of the ice-cake, but showing traces in its shells and its stratified layers of the presence of the sea. There on the surface of the rock the boulder-clay has rested throughout the long period during which the land subsided to a depth of 1500 or 1700 feet or more, and again re-emerged till it approached, under an ever softening climate and with a more temperate fauna, to almost its present level.

In short, the 40-foot beach terrace owes its configuration not so much to the external action of the sea, as to the internal contour of the rock. That contour was derived from the grinding agency of the ice-cake covering, and between the period when that contour was given to it, and the presence of the sea at the 40-foot beach-line, there have intervened all the countless ages, with their manifold changes, of the subsidence and re-elevation of our land.

The following gentlemen were elected Fellows of the Society:—

ARTHUR ABNEY WALKER, Esq.
JOHN FOULERTON, M.D.

The following Donations to the Library were announced:—

Memorie della reale Accademia delle scienze di Torino. Serie seconda: tomo XX. 4to.—*From the Academy.*

Proceedings of the Royal Horticultural Society, May 1864. 8vo. —*From the Society.*

Nova Acta Academiæ Cæsareæ Leopoldino-Carolinæ Germanicæ Naturæ Curiosorum. Tomus Tricesimus. 4to.—*From the Academy.*

Abhandlungen der königl. Gesellschaft der Wissenschaften zu Göttingen. XIter Band. von den Jahren 1862 und 1863. 4to.—*From the Society.*

Nachrichten von der Georg-Augusts-Universität zu Göttingen. Nr. 1–21. Nebst Register. 8vo.—*From the University.*

Mémoires de la Société de Physique et d'Histoire naturelle de

Genève. Tome XVII. Première Partie. 4to.—*From the Society.*

Annalen der königl. Sternwarte bei München IV. Supplement band. 8vo.—*From the Observatory.*

Scheikundige Verhandelingen en onderzoekingen uitgegeven door G. J. Mulder. Derde Deel—derde stuk. 8vo.—*From the Author.*

Journal of the Scottish Meteorological Society, April 1864. 8vo. —*From the Society.*

Proceedings of the Academy of Nat. Sciences of Philadelphia. Nos. 3–7. 1863. 8vo.—*From the Academy.*

Observations on the Genus Unio. By Isaac Lea, LL.D. Vol. X. 4to.—*From the Author.*

Journal of the Academy of Natural Sciences of Philadelphia. Vol. V. Part IV. 4to.—*From the Academy.*

Yorkshire Philosophical Society Annual Report for 1863. 8vo. —*From the Society.*

Astronomical and Meteorological Observations made at the U.S. Naval Observatory during 1862. 4to.—*From the U.S. Naval Observatory.*

Abstracts of Magnetical Observations made at the Magnetical Observatory, Toronto, during 1856–62, and during parts of 1853, 1854, and 1855. 4to.—*From the Observatory.*

Observations of the Spots on the Sun from November 9, 1853 to March 24, 1861, made at Redhill by Richard Christopher Carrington, F.R.S. 4to.—*From the Author.*

Jahresbericht über die Fortschritte der Chemie herausgegeben von H. Kopp u. H. Will, für 1862. Zweites Heft. 8vo.—*From the Authors.*

Comptes Rendus for 1863. 4to.—*From the Academy of Sciences.*

Journal of the Society of Arts for 1863. 8vo.—*From the Society.*

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